Technical Report on Fatigue Properties — SAE J1099

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TECHNICAL REPORT ON FATIGUE PROPERTIES — SAE J1099

Report of the Fatigue Design and Evaluation Steering Committee approved February 1975

1. SCOPE

1.1 Pertinent information to provide design guidance in avoiding fatigue failures is outlined in this SAE Information Report. Of necessity, it is brief, but it does provide a basis for approaching complex fatigue problems. The information provided here can be used in preliminary design estimates of fatigue life, the selection of materials and the analysis of service load and/or strain data.

2. INTRODUCTION

2.1 Designing to avoid fatigue failures of a component of a vehicle (or machine or structure) is one of the more difficult tasks an engineer faces. Many factors are involved with the relationships between the factors only partially established and largely empirical. Fatigue failure is caused by repeated loading with the number of cycles of loading to failure varying with the load range. "Damage" accrues gradually over the life span of cyclical loading, with small changes taking place during the total life. Crack initiation occurs at varying percentages of the final life, and crack propagation continues until final fracture takes place.

2.2 Designing to avoid fatigue failures requires knowledge of the following:

2.2.1 The expected load-time history (or better, the local stress-time and strain-time history at the most critical locations).

2.2.2 The nature of the environment in which the component is operated (wet, dry, corrosive, temperature, etc.)

2.2.3 The properties of the material as it exists in the finished component at the most critically stressed locations ("inherent" fatigue properties, residual stress effects, surface effects, sensitivity to corrosion, "cleanliness," variability, etc.).

2.2.4 The geometry of the component and its notches (stress concentrations, surface finish, manufacturing variability, etc.)

2.3 Scatter in fatigue life is another aspect of fatigue life evaluation and prediction which must be considered. This often calls for statistically based analyses. Circumstances dictate the degree of sophistication required in all aspects of an evaluation or prediction.

2.4 In the decades that have followed the first recorded observation of the fatigue phenomenon, well over 100 years ago, much has been done to enable progressively better fatigue life predictions. The concept of the fatigue limit — the stress level below which fatigue failure is highly improbable — has been used extensively, probably because it is simple to apply and because it has an empirical relationship to the ultimate tensile strength.

2.5 Early in automotive history, the "proving ground approach" to evaluating performance of "systems" came into being. Weak components failed and had to be strengthened; it was that simple. System tests of this kind still play a predominant role in the industry and are used together with laboratory component and system testing today. A vast amount of specific experience and data have been acquired in this way, permitting comparative testing, in which a new component or system is compared with those made previously. This provides another valuable approach to the problem of measuring fatigue performance and some guidance in design. More general approaches are necessary, however, for many applications.

2.6 In the past 15 - 20 years, substantial advances have been made in the understanding of fatigue phenomenon, design engineering concepts and testing methods and equipment. Considerable attention has been devoted to the subject of fracture mechanics. In many design situations it will be necessary to consider the mechanism of crack propagation and fracture as well as the initiation of the crack. Fracture mechanics may be suited to the task of predicting the life of a cracked component. The yield strength of a material determines the minimum size crack to which fracture mechanics can be applied. With the proper information, the rate at which a given crack will grow slowly to a specific size as well as the size at which it will propagate catastrophically can be estimated. Also, instances when a given crack will not propagate can be estimated. At the present time, only a small quantity of data on fracture mechanics properties of steels used by the ground vehicle industry have been published. As larger amounts of data become available, definitions and tables will be added to this report. More information on this subject is given in Refs. 5 - 11.

3. MATERIAL PROPERTY TABLES

3.1 Table 1 is a listing of monotonic stress-strain properties for selected metals. Table 2 is a listing of cyclic and fatigue properties. Steels are listed first, followed by the aluminums. Within these broad classifications, the metals are listed in order of SAE specification then increasing true fracture strength. Table 3 lists the non-standard abbreviations necessitated by computer printed tables in this report. A brief introduction, definitions and discussion follow the tables. An example application is given at the end of the report.

3.2 Material properties given in the current tables represent only the beginning for this report. In order that the quantity and quality of this data may grow, it is necessary that additional information be contributed to the existing data bank. As data become available, this report will be updated. Persons wishing to contribute may contact the Fatigue Design & Evaluation Committee through the SAE Detroit Branch Office, 18121 East Eight Mile Road, East Detroit, Michigan 48021.

3.3 The majority of the properties listed in the tables have been contributed by members of the SAE Fatigue Design & Evaluation Committee. Values listed in the tables were obtained by testing a sample of the metal specified. The size of the sample varied from a very few test bars cut from a single sample of the metal to numerous bars from more than one source. A data reference number is given to identify the original source of the properties. As defined, these properties are measured on smooth polished specimens and therefore do not include such influences as environmental effects (wet or corrosive conditions, elevated temperature, etc.), surface roughness effects, mean or residual stress effects, notch effects, etc.

3.4 There are many procedures for using this information for the above mentioned purposes. They are too lengthy to be included in this report; however, there are a number of publications which discuss these procedures. One of them is the SAE Fatigue Design Handbook (1)* which discusses fatigue properties, methods of determining fatigue properties, and illustrates the use of this data for making design decisions. Other useful references are listed at the end of this report.

TABLE 3 — LIST OF NON-STANDARD ABBREVIATIONS OF MATERIAL PROPERTY NAMES

CYC STRAIN HARD'G EXP, n'	Cyclic Strain Hardening Exponent
CYC STR COF, K'	Cyclic Strength Coefficient
CYC YLD, S_{ys}, σ_{ys}	Cyclic Yield Strength
DIR	Direction
FAT DUC COF, ϵ'_f	Fatigue Ductility Coefficient
FAT DUC EXP, c	Fatigue Ductility Exponent
FAT STR COF, σ'_f	Fatigue Strength Coefficient
FAT STR EXP, b	Fatigue Strength Exponent
L	Longitudinal Grain Direction
LT	Long Transverse Grain Direction
MOD OF ELAS, E	Modulus of Elasticity
STRAIN HARD'G EXP, n	Strain Hardening Exponent
STR COF, K	Strength Coefficient
TRUE FRACT DUCT, ϵ_f	True Fracture Ductility
TRUE FRACT STR, σ_f	True Fracture Strength
ULT STR, S_u	Ultimate Strength
YIELD STR, S_{ys}, σ_{ys}	Yield Strength
% RA	Percent Reduction of Area

4. MONOTONIC STRESS-STRAIN PROPERTIES

4.1 Monotonic* stress-strain properties are generally determined by testing a smooth polished specimen under axial loading.

4.2 The Load, diameter and/or strain on the uniform test section is measured during the test in order to determine the materials stress-strain response as illustrated in Figure 1 and 2. Properties, most of which are discrete points on the stress-strain curve, can be defined to describe the behavior of a material

4.3 DEFINITIONS:

4.3.1 Ultimate Tensile Strength (S_u) is the engineering stress at maximum load. In a ductile material it is governed by necking of the specimen.

$$S_u = P_{max}/A_o \quad (1)$$

where P_{max} = maximum load

A_o = original cross sectional area

4.3.2 True Fracture Strength (σ_f) is the "true" tensile stress required to cause fracture.

$$\sigma_f = P_f/A_f \quad (2)$$

where P_f = load at failure

A_f = minimum cross sectional area after failure

The value must be corrected for the effect of triaxial stress present due to necking. One such correction suggested by Bridgman (2) is illustrated in Figure 3. In this figure, the ratio of the corrected value to the uncorrected value ($\sigma_f/(P_f/A_f)$) is plotted against true tensile strain.

4.3.3 Tensile Yield Strength (S_{ys},σ_{ys}) is the stress to cause a specified amount of inelastic strain, usually 0.2%. It is usually determined by constructing a line of slope E through 0.2% strain and zero stress. The stress where the constructed line intercepts the stress-strain curve is taken as the yield strength (E = modulus of elasticity, see Fig. 1).

4.3.4 Percent Reduction of Area (% RA) is the percentage of reduction in cross sectional area after fracture.

$$\%\,RA = 100\ \frac{A_o - A_f}{A_o} \quad (3)$$

4.3.5 True Fracture Ductility (ϵ_f) is the "true" plastic strain after fracture.

$$\epsilon_f = \ln(A_o/A_f) = \ln(100/(100\text{-}\%RA)) \quad (4)$$

4.3.6 Monotonic Strain Hardening Exponent (n) is the power to which the "true" plastic strain must be raised to be proportional to "true" stress. It is generally taken as the slope of log σ_p versus log ϵ_p plot as shown in Fig. 2.

$$\sigma = K\epsilon_p^n \quad (5)$$

4.3.7 Monotonic Strength Coefficient (K) is the "true" stress at a "true" plastic strain of unity as

*Numbers in parenthesis refer to References listed at the end of the report.

*The term monotonic is used in preference to static since the test is usually conducted by continuously increasing with time the distance between the cross heads of the test machine (or better still, the strain on the specimen) until fracture occurs.

shown in Fig. 2. If fracture ductility is less than 1.0, it is necessary to extrapolate. (see Eq. 5)

4.4 DISCUSSION

4.4.1 Monotonic tension properties of a material can be classed into two groups, engineering stress-strain properties and "true" stress-strain properties. Engineering properties are associated with the original cross sectional area of the test specimen and "true" values relate to actual area while the specimen is under load. The difference between "true" and engineering values is insignificant in the low strain region, up to 1 - 2% strain.

4.4.2 Until the test bar begins to locally neck, some simple relationships exist between engineering and "true" stress-strain values. Eq. 6 gives the relationship between engineering and true strain.

$$\epsilon = \ln(1 + e) \qquad (6)$$

where ϵ = "true" strain

ϵ = engineering strain

Similarly, Eq. 7 relates true stress to engineering stress.

$$\sigma = S(1 + e) \qquad (7)$$

where σ = "true" stress

S = engineering stress

4.4.3 A more detailed discussion and derivation of monotonic stress-strain properties can be found in ASTM STP 465 (3). Figs. 1 and 2 graphically illustrate a majority of these properties.

5. CYCLIC STRESS-STRAIN PROPERTIES

5.1 Cyclic stress-strain properties are determined by testing smooth polished specimens under axial cyclic strain control. The cyclic stress-strain curve is defined as the locus of tips of stable "true" stress-strain hyteresis loops obtained from companion test specimens. A typical stable hysteresis loop is illustrated in Fig. 4 and a set of stable loops with a cyclic stress-strain curve down through the loop tips is illustrated in Fig. 5. As illustrated, the height of the loop from tip-to-tip is defined as the stress range ($\Delta\sigma$). For completely reversed testing one-half of the stress range is generally equal to the stress amplitude while one-half of the width from tip-to-tip is defined as the strain amplitude ($\Delta\epsilon/2$). Plastic strain amplitude is found by subtracting the elastic strain amplitude ($\Delta\epsilon_e/2$) from the strain amplitude as indicated in Eqs. 8-10.

$$\Delta\epsilon_p/2 = \Delta\epsilon/2 - \Delta\epsilon_e/2 \qquad (8)$$

According to Hooke's law,

$$\Delta\epsilon_e/2 = \Delta\sigma/2E \qquad (9)$$

where E - modulus of elasticity,

$$\Delta\epsilon_p/2 = \Delta\epsilon/2 - \Delta\sigma/2E \qquad (10)$$

5.2 A more complete discussion of the cyclic stress-strain curve and other methods of obtaining the curve are given in STP 465 (3) and in Ref. 4

5.3 DEFINITIONS

5.3.1 Cyclic Yield Strength (0.2% σ_{ys}) is the stress to cause 0.2% inelastic strain as measured on a cyclic stress-strain curve. It is usually determined by constructing a line parallel to the slope of the cyclic stress-strain curve at zero stress through 0.2% strain and zero stress. The stress where the constructed line intercepts the cyclic stress-strain curve is taken as the 0.2% cyclic yield strength.

5.3.2 Cyclic Strain Hardening Exponent (n') is the power to which "true" plastic strain amplitude must be raised to be proportional to "true" stress amplitude. It is taken as the slope of the log $\Delta\epsilon_p/2$ and $\Delta\sigma/2$ versus log $\Delta\sigma/2$ plot, were $\Delta\epsilon_p/2$ and $\Delta\sigma/2$ are measured from cyclically stable hysteresis loops.

$$\Delta\sigma/2 = K'(\Delta\epsilon_p/2)^{n'} \qquad (11)$$

where $\Delta\epsilon_p/2$ - "true" plastic strain amplitude

The line defined by this equation is illustrated in Fig. 6.

5.3.3 Cyclic Strength Coefficient (K') is the "true" stress at a "true" plastic strain of unity in Eq. 11. It may be necessary to extrapolate as indicated in Fig. 6.

5.4 DISCUSSION

5.4.1 Stress-strain response of some steels can change significantly when subjected to inelastic strains such as can occur at notch roots due to cyclic loading. When fatigue failure occurs, particularly low cycle fatigue, such inelastic straining is generally present. Hence, the cyclic stress-strain curve may better represent the steel's stress-strain response than the monotonic stress-strain curve.

5.4.2 In many field test situations it may be desirable to convert measured strains to stress in order to estimate fatigue life. The cyclic stress-strain curve can be described with an equation using the cyclic properties. Eq. 10 can be rewritten rearranging the terms as shown in Eq. 12.

$$\Delta\epsilon/2 = \Delta\sigma/2E + \Delta\epsilon_p/2 \qquad (12)$$

Rearranging the terms in Eq. 11 indicates the relationship between plastic strain amplitude and stress amplitude.

$$\Delta\epsilon_p/2 = (\Delta\sigma/2K')^{1/n} \qquad (13)$$

Substituting Eq. 13 into Eq. 12 yields an equation relating cyclic strain amplitude to cyclic stress amplitude in terms of the previously defined properties and the modulus of elasticity.

$$\Delta\epsilon/2 = \Delta\sigma/2E + (\Delta\sigma/2K')^{1/n'} \qquad (14)$$

5.4.3 For a more detailed discussion see STP 465 (3).

6. FATIGUE PROPERTIES

6.1 Fatigue resistance of metals is generally described in terms of the number of constant amplitude stress or strain reversals* required to cause failure. The properties defined in this section are determined on smooth polished axial specimens tested under strain control. Stress amplitude, strain amplitude and plastic strain amplitude can each be plotted against reversals to failure. The plot of log "true" plastic strain amplitude versus log reversals to failure are typically straight lines as illustrated in Figs. 7 and 8. The intercept at one reversal and the slope of these straight lines can be described as fatigue properties.

6.2 DEFINITIONS

6.2.1 Fatigue Ductility Exponent (c) is the power to which the life in reversals must be raised to be proportional to the "true" strain amplitude. It is taken as the slope of the log ($\Delta\epsilon_p/2$) versus log ($2N_f$) plot.

6.2.2 Fatigue Ductility Coefficient (ϵ'_f) is the "true" strain required to cause failure in one reversal. It is taken as the intercept of the log ($\Delta\epsilon_p/2$) versus log ($2N_f$) plot at $2N_f = 1$.

6.2.3 Fatigue Strength Exponent (b) is the power to which life in reversals must be raised to be proportional to "true" stress amplitude. It is taken as the slope of the log $\Delta\sigma/2$ versus log ($2N_f$) plot.

6.2.4 Fatigue Strength Coefficient (σ_f') is the "true" stress required to cause failure in one reversal. It is taken as the intercept of the log $\Delta\sigma/2$ versus log ($2N_f$) plot at $2N_f = 1$.

6.2.5 Transition Fatigue Life ($2N_t$) is the life where elastic and plastic components of the total strain are equal. It is the life at which the plastic and elastic strain-life lines cross.

6.3 DISCUSSION

6.3.1 A metals resistance to strain cycling can be considered as the summation of the elastic and plastic resistance as indicated by Eq. 15.

$$\Delta\epsilon/2 = (\Delta\epsilon_e/2) + (\Delta\epsilon_p/2) \qquad (15)$$

An equation of the "true" plastic strain-life relationship can be written in terms of the above fatigue properties (Fig. 8).

$$\Delta\epsilon_p/2 = \epsilon'_f\,(2N_f)^c \qquad (16)$$

where $2N_f$ is reversal to failure. The "true" elastic strain-life relationship is simply the stress-life relationship divided by the modulus of elasticity (Fig. 7).

$$\Delta\epsilon_e/2 = (\sigma'_f/E\,(2N_f)^b \qquad (17)$$

Substituting Eq. 16 and Eq. 17 into Eq. 15 gives an equation between "true" strain amplitude and reversals to failure in terms of the fatigue properties.

$$\Delta\epsilon/2 = (\sigma'_f/E)\,(2N_f)^b + \epsilon'_f\,(2N_f)^c \qquad (18)$$

The above equation is illustrated in Fig. 9.

7. ILLUSTRATIVE EXAMPLE

7.1 The purpose of this section is to illustrate some of the possible applications of cyclic stress-strain and fatigue properties. One example is presented illustrating how the data in this report might be utilized. It by no means covers the many possible ways in which this data can or will be used in design considerations. Other writings such as Sandor's recent book (17) and other references cover this subject in much greater detail.

7.2 Consider the problem of estimating fatigue life for a given strain history. Three different metals will be considered: 1020 hot rolled (HR) steel, 1045 quenched and tempered (Q&T) steel at 390 BHN, and 2024-T4 aluminum. A strain-time history at the point of possible failure is given in the following sequence of peaks and valleys: $e_o = 0.0, e_1 = 0.0045$, $e_2 = -0.002, e_3 = 0.004, e_4 = -0.0045, e_5 = 0.003, e_6 = -0.0045$, and $e_7 = 0$. (see Fig. 10.) To simplify the example, it is assumed that the strains given include the influence of any stress concentration that may be present. For example, they might represent the strains at a notch root. Further details concerning notch effects and a method for converting applied loads or strains measured near a notch to notch root strains can be found in Refs. 12-14.

7.3 First, a check should be made to determine if yielding takes place during the first block of straining. This is done by comparing the monotonic yield strength with the largest strain multiplied by the modulus of elasticity. Since yielding may occur in compression as well as tension, the peak or valley with the largest absolute value should be used in the comparison. (see Table 4.) Clearly, the 1020 yields while the 1045 remains elastic at least for the first block of load. The 2024 yields a small amount.

TABLE 4

SAE Spec.	Modulus of Elasticity, E, ksi	$e_{max} \cdot E$, ksi	Monotonic Yield Strength ksi
1020 HR	29,500	133	38
1045 Q&T	30,000	135	185
2024-T4	10,200	46	44

TABLE 5

SAE Spec.	Monotonic Strain Hardening Exponent, n	Cyclic Strain Hardening Exponent, n
1020 HR	0.19	0.18
1045 Q&T	0.044	0.17
2024-T4	0.20	0.08

7.4 The next question to ask is whether the chosen metals will cyclically harden or soften. That is, will the metals stress-strain response change during cycling such that yielding may occur later or possibly stop? The first indication can be obtained by comparing the monotonic and cyclic strain hardening exponents of the metals. If the cyclic is less than the monotonic strain hardening exponent, the metal will

*A reversal is counted each time the stress or strain-time signal changes direction. In constant amplitude testing, one cycle is equal to two reversals.

generally cyclically harden. If it is greater it will usually cyclically soften. (see Table 5.)

7.5 Little change should be expected for the 1020 HR, considerable cyclic softening for the 1045 Q&T, and considerable cyclic hardening for the 2024. A check can be made to see if yielding occurs after the metal has been cycled for some time by comparing twice the cyclic yield strength to the largest strain range in the history multiplied by the modulus of elasticity. For this comparison, the largest strain range is defined as the maximum strain minus the minimum strain, regardless of whether they occur adjacent in the load sequence or not (Table 6).

7.6 The 1020 HR is still in a yield condition; 1045 Q&T is now yielding, whereas it did not yield in the first reversal; and the 2024 T4 is no longer yielding. Note, that if the question as to whether yeild would occur has been based upon the monotonic value, an incorrect answer would have been arrived at in two out of the three choices.

7.7 Most important is the consideration of fatigue life of the component. Life can be estimated from the strain data alone. For simplicity, the effect of mean stress will be ignored here, although it may be an influential and even the deciding factor in a more detailed analysis. The first step is to separate the strain signal into individual cycles via a cycle counting routine. Numerous methods for counting have been suggested. However, the most promising is the rainflow method (15). This method can be shown to separate the history into stress-strain hysteresis loops which are comparable to those found in the constant amplitude material properties testing. As illustrated in Fig. 10, rainflow counting breaks this service history into three closed loops: 0.0045 to -0.0045, 0.004 to -0.002, and -0.0045 to 0.0025 strain.

TABLE 6

SAE Spec.	Modulus of Elasticity, E, ksi	$\Delta e_{max} \cdot E$, ksi	2 • Cyclic Yield Strength ksi
1020 HR	29,500	266	70
1045 Q&T	30,000	270	220
2024-T4	10,200	92	128

7.8 Fatigue life will be estimated using a linear damage rule:

$$\text{Fatigue life in blocks to failure} = \frac{1}{\Sigma \frac{n}{N_f}} \qquad (19)$$

where:

Block = one time through strain sequence

n = cycles counted in sequence at given amplitude

N_f = cycles to crack initiation

7.9 The life, N_f, is found by substituting the four fatigue properties and the modulus of elasticity as given in Table 2 into Eq. 18 (Table 7):

$$\text{for 1020 HR } \frac{\Delta\epsilon}{2} = \frac{130}{29{,}500} (2N_f)^{-0.12} + 0.41 (2N_f)^{-0.51}$$

$$\text{for 1045 Q\&T } \frac{\Delta\epsilon}{2} = \frac{230}{30{,}000} (2N_f)^{-0.074} + 0.45 (2N_f)^{-0.68}$$

$$\text{for 2024 T4 } \frac{\Delta\epsilon}{2} = \frac{147}{10{,}200} (2N_f)^{-0.11} + 0.21 (2N_f)^{-0.52}$$

7.10 Cycles-to-failure, N_f, for the strain ranges counted, found by solving the above equations, is given in Table 8.

TABLE 7

SAE Spec.	E, Modulus of Elasticity, ksi	σ'_f, Fatigue Strength Coefficient, ksi	b, Fatigue Strength Exponent	ϵ'_f, Fatigue Ductility Coefficient	c, Fatigue Ductility Exponent
1020 HR	29,500	130	-0.12	0.41	-0.51
1045 Q&T	30,000	230	-0.074	0.45	-0.68
2024-T4	10,200	147	-0.11	0.21	-0.52

TABLE 8

Hysteresis Loop	Strain Range	Fatigue Life, N_f 1020HR	1045Q&T	2024-T4
1-6	0.0090	7,100	6,700	57,000
4-5	0.0070	14,000	43,500	330,000
2-3	0.0060	21,500	215,000	1,090,000

7.11 For the given condition of equal strain histories, it is clear that the 2024-T4 would have the longest fatigue life. The above prediction is strain based and does not account for any mean stress effects. Also, the load-carrying capacity of an aluminum member would be smaller than for the steel members because of the differences in elastic modulus.

7.12 In order to include mean stress, the stresses which correspond to the given strains must be determined. Since inelastic material behavior is occurring, it will be necessary to follow cyclic stress-strain response instead of simply multiplying strain times modulus of elasticity to obtain stress.

TABLE 9

SAE Spec.	E, Modulus of Elasticity ksi	n', Cyclic Strain Hardening Exponent	K', Cyclic Strength Coefficient, ksi
1020 HR	29,500	0.18	112
1045 Q&T	30,000	0.17	263
2024-T4	10,200	0.08	166

7.13 To simplify the analysis, since hardening or softening generally takes place in the early portion of the life, the stress-strain response will be assumed to be equal to the stable cyclic condition. Stable cyclic response is described by Eq. 14 involving the modulus of elasticity, cyclic strain hardening exponent, and cyclic strength coefficient. These properties are listed in Table 2 except for the cyclic strength coefficient of 1045 Q&T and 2024-T4. In the case where the cyclic strength coefficient is not reported, it can be estimated knowing the fatigue strength and ductility coefficient and the cyclic strain hardening exponent. The cyclic strength and ductility coefficient represent a point on the plastic strain amplitude versus stress amplitude plot and the cyclic strain hardening exponent is the slope of the line. The cyclic strength coefficient can be calculated from Eq. 11 as follows:

$$\Delta\sigma/2 = K'(\Delta\epsilon_p/2)^{n'}$$

$$K' = (\Delta\sigma/2)/(\Delta\epsilon_p/2)^{n'}$$

$$K' = \sigma'_f/(\epsilon'_f)^{n'}$$

* See Table 9

7.14 For a repeated sequence, the first half cycle is unique starting at zero stress and zero strain; for this example, it is assumed to follow the stable cyclic stress-strain response as indicated in Eq. 20.

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} \qquad (20)$$

7.15 The next reversal follows the stress-strain outer loop curve. The outer loop shape has been found to be different from the cyclic curve by approximately a factor of two (16). Hence, the outer loop equation is as follows:

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \qquad (21)$$

7.16 Substituting the properties from Table 9 into Eq. 21 the stress range for each strain range may be calculated, as shown in Table 10.

7.17 Starting with the stress determined by following the cyclic stress-strain curve to the first point in the history, the above ranges can be algebraically added to each succeeding point to determine the sequential stress peaks. With stable cyclic response, each block of the strain sequence will result in a repeating stress sequence. The repeating sequence for the three materials is indicated in Table 11.

TABLE 10

Hysteresis Loop Tips*	Strain Range	Stress Range, ksi 2024-T4	1045 Q&T	1020 HR
1-2	0.0065	66.3	153.0	73.2
2-3	0.006	61.2	147.0	71.7
1-4	0.009	91.8	175.6	79.4
4-5	0.007	71.4	158.5	74.7

* Associated hysteresis loop tips determined by rainflow counting.

TABLE 11

Sequence Point	Strain Sequence	Stable Cyclic Stress Sequence, ksi 2024-T4	1045 Q&T	1020 HR
1	0.0045	45.9	87.8	39.7
2	-0.002	-20.4	-65.2	-33.5
3	0.004	40.8	81.8	38.2
4	-0.0045	-45.9	87.8	-39.7
5	0.0025	25.5	70.7	35.0
6	-0.0045	-45.9	-87.8	-39.7

TABLE 12

Hysteresis Loop	Strain Range	Mean Stress, ksi 2024-T4	1045 Q&T	1020 HR
1-6	0.009	0.0	0.0	0.0
2-3	0.006	10.2	8.3	2.35
4-5	0.007	-10.2	-8.55	-2.35

TABLE 13

Hysteresis Loop	Fatigue Life N_f 2024-T4	1045 Q&T	1020 HR
1-6	57,000	6,700	7,100
2-3	635,000	145,000	21,000
4-5	535,000	60,500	14,200

7.18 Rainflow counting identifies the cycles as three closed loops with tips at points 1 and 6, 2 and 3, and 4 and 5. The strain range and mean stress of each cycle is tabulated in Table 12.

7.19 Numerous methods have been suggested to account for mean stress effect. For this example, a method suggested by Morrow (1) will be used. This results in Eq. 18 being modified as shown below:

$$\frac{\Delta\epsilon}{2} = \frac{\sigma'_f - \sigma_o}{E}(2N_f)^b + \epsilon'_f(2N_f)^c \qquad (22)$$

7.20 Cycles-to-failure is found by solving Eq. 22 for the combinations of mean stress and strain ranges from Table 12. This is shown in Table 13. Fatigue life for each metal is estimated below:

$$\text{Life 1020 HR} = \frac{1}{1/7{,}100 + 1/21{,}000 + 1/14{,}200} = 3862 \text{ blocks}$$

$$\text{Life 1045 Q+T} = \frac{1}{1/6{,}700 + 1/145{,}000 + 1/60{,}500} = 5790 \text{ blocks}$$

$$\text{Life 2024-T4 aluminum} = \frac{1}{1/57{,}000 + 1/635{,}000 + 1/535{,}000} = 47{,}600 \text{ blocks}$$

7.21 The addition of a mean stress correction in this example had little influence on the resulting estimates of life. This is generally true when the damage is principally due to a few large cycles-low cycle fatigue. In cases where failure is due to a larger number of small cycles, mean stress may have a much greater influence.

8. REFERENCES

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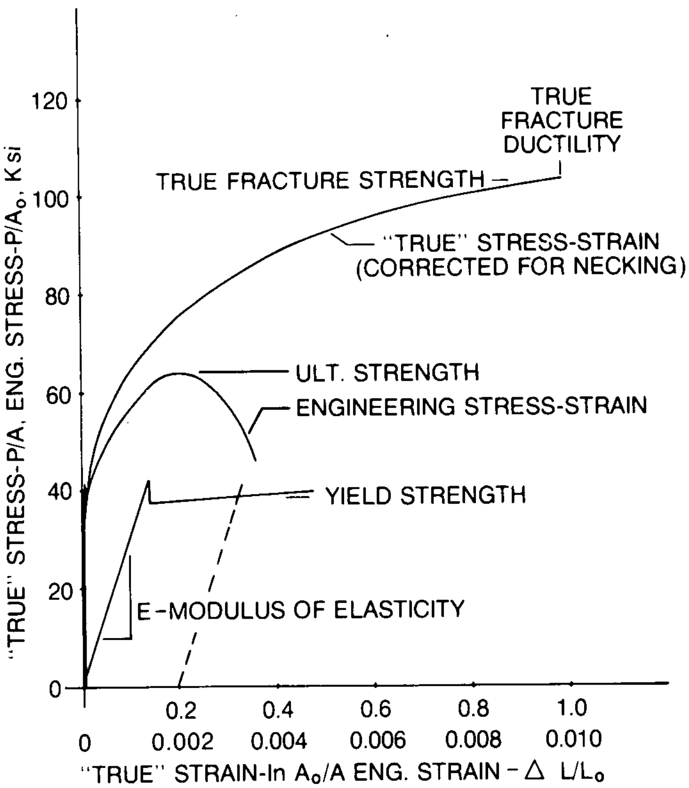

FIG. 1 - ENGINEERING AND "TRUE" STRESS PLOT, 1020 HR STEEL

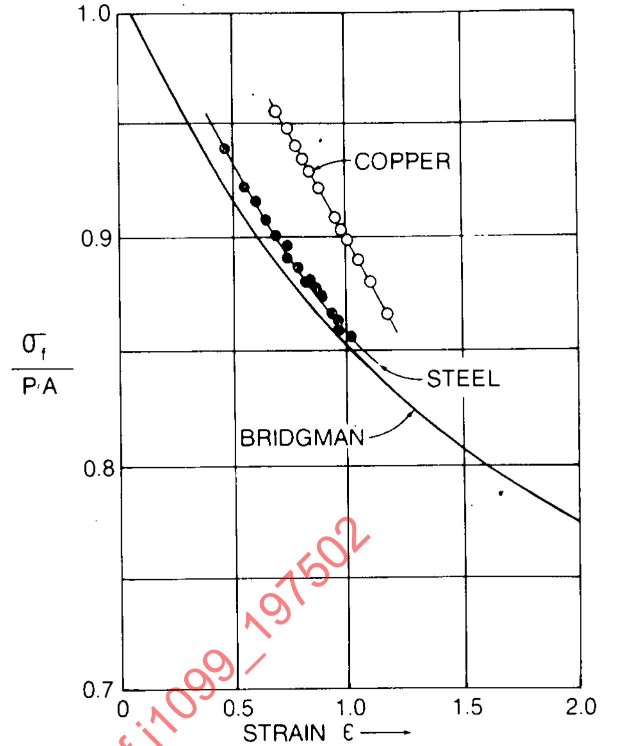

FIG. 3 - RELATIONSHIP BETWEEN BRIDGMAN CORRECTION FACTOR $\frac{\sigma_f}{P/A}$ AND TRUE TENSILE STRAIN

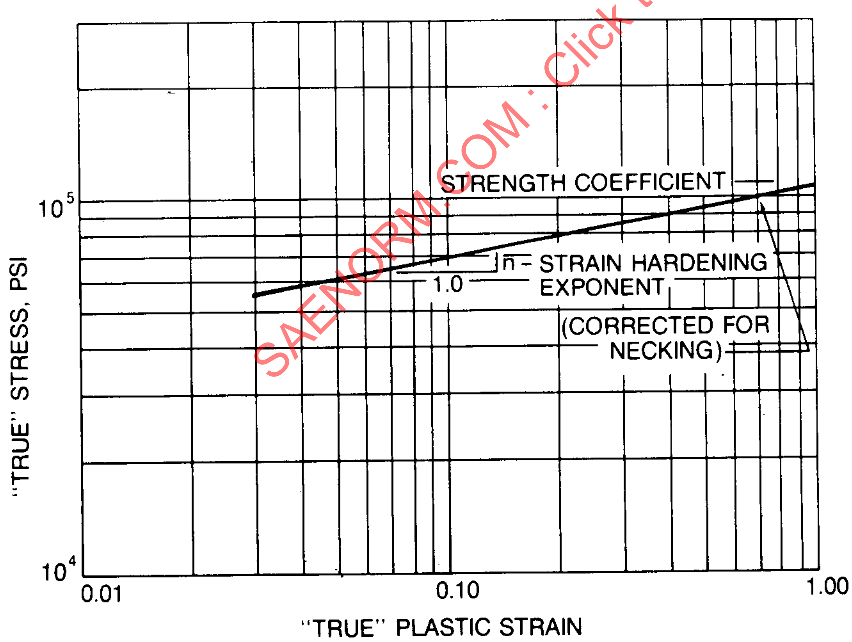

FIG. 2 - "TRUE" STRESS-PLASTIC STRAIN PLOT, 1020 HR STEEL

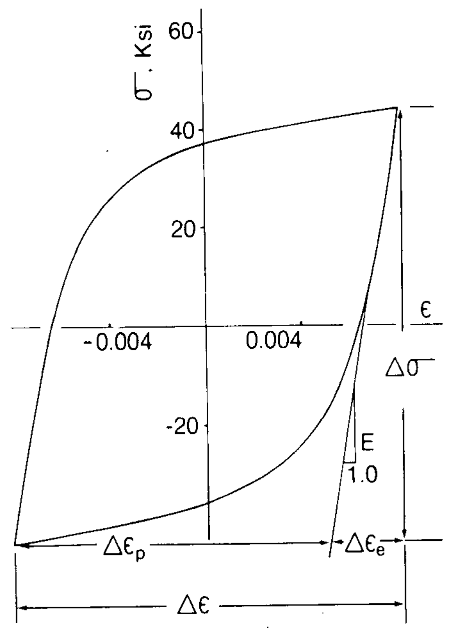

FIG. 4 - STABLE STRESS-STRAIN HYSTERESIS LOOP