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**Information technology — Security  
techniques — Digital signature schemes  
giving message recovery —**

**Part 2:  
Integer factorization based mechanisms**

*Technologies de l'information — Techniques de sécurité — Schémas de  
signature numérique rétablissant le message —*

*Partie 2: Mécanismes basés sur une factorisation entière*

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## Contents

Page

|   |           |
|---|-----------|
| <b>Foreword</b> .....   | <b>v</b>  |
| <b>Introduction</b> .....   | <b>vi</b> |
| <b>1 Scope</b> .....  | <b>1</b>  |
| <b>2 Normative references</b> .....   | <b>1</b>  |
| <b>3 Terms and definitions</b> .....  | <b>1</b>  |
| <b>4 Symbols and abbreviated terms</b> .....  | <b>3</b>  |
| <b>5 Converting between bit strings and integers</b> .....  | <b>5</b>  |
| <b>6 Requirements</b> .....   | <b>5</b>  |
| <b>7 Model for signature and verification processes</b> .....   | <b>6</b>  |
| <b>7.1 Signing a message</b> .....  | <b>7</b>  |
| <b>7.1.1 Overview</b> .....   | <b>7</b>  |
| <b>7.1.2 Message allocation</b> .....   | <b>7</b>  |
| <b>7.1.3 Message representative production</b> .....  | <b>7</b>  |
| <b>7.1.4 Signature production</b> .....   | <b>7</b>  |
| <b>7.2 Verifying a signature</b> .....  | <b>8</b>  |
| <b>7.2.1 Overview</b> .....   | <b>8</b>  |
| <b>7.2.2 Signature opening</b> .....  | <b>8</b>  |
| <b>7.2.3 Message recovery</b> .....   | <b>8</b>  |
| <b>7.2.4 Message assembly</b> .....   | <b>8</b>  |
| <b>7.3 Specifying a signature scheme</b> .....  | <b>8</b>  |
| <b>8 Digital signature scheme 1</b> .....   | <b>9</b>  |
| <b>8.1 Parameters</b> .....   | <b>9</b>  |
| <b>8.1.1 Modulus length</b> .....   | <b>9</b>  |
| <b>8.1.2 Trailer field options</b> .....  | <b>9</b>  |
| <b>8.1.3 Capacity</b> .....   | <b>9</b>  |
| <b>8.2 Message representative production</b> .....  | <b>9</b>  |
| <b>8.2.1 Hashing the message</b> .....  | <b>9</b>  |
| <b>8.2.2 Formatting</b> .....   | <b>9</b>  |
| <b>8.3 Message recovery</b> .....   | <b>10</b> |
| <b>9 Digital signature scheme 2</b> .....   | <b>11</b> |
| <b>9.1 Parameters</b> .....   | <b>11</b> |
| <b>9.1.1 Modulus length</b> .....   | <b>11</b> |
| <b>9.1.2 Salt length</b> .....  | <b>11</b> |
| <b>9.1.3 Trailer field options</b> .....  | <b>11</b> |
| <b>9.1.4 Capacity</b> .....   | <b>12</b> |
| <b>9.2 Message representative production</b> .....  | <b>12</b> |
| <b>9.2.1 Hashing the message</b> .....  | <b>12</b> |
| <b>9.2.2 Formatting</b> .....   | <b>12</b> |
| <b>9.3 Message recovery</b> .....   | <b>12</b> |
| <b>10 Digital signature scheme 3</b> .....  | <b>13</b> |
| <b>Annex A (normative) Public key system for digital signature</b> .....  | <b>14</b> |
| <b>Annex B (normative) Mask generation function</b> .....   | <b>18</b> |
| <b>Annex C (informative) On hash-function identifiers and the choice of the recoverable length of the message</b> ..... | <b>20</b> |
| <b>Annex D (informative) Examples</b> .....   | <b>21</b> |
| <b>Bibliography</b> .....   | <b>47</b> |

## Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work. In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

The main task of the joint technical committee is to prepare International Standards. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

ISO/IEC 9796-2 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC 27, *IT Security techniques*.

This second edition cancels and replaces the first edition (ISO/IEC 9796-2:1997), which has been technically revised. Implementations which comply with ISO/IEC 9796-2 (1st edition), and which use a hash-code of at least 160 bits in length, will be compliant with ISO/IEC 9796-2 (2nd edition). Note, however, that implementations complying with ISO/IEC 9796-2 (1st edition) that use a hash-code of less than 160 bits in length will not be compliant with ISO/IEC 9796-2 (2nd edition).

ISO/IEC 9796 consists of the following parts, under the general title *Information technology — Security techniques — Digital signature schemes giving message recovery*:

- *Part 1: Mechanisms using redundancy*
- *Part 2: Integer factorization based mechanisms*
- *Part 3: Discrete logarithm based mechanisms*

Further parts may follow.

Annexes A and B form a normative part of this part of ISO/IEC 9796. Annexes C and D are for information only.

## Introduction

Digital signature mechanisms can be used to provide services such as entity authentication, data origin authentication, non-repudiation, and integrity of data. A digital signature mechanism satisfies the following requirements.

- Given the verification key but not the signature key it shall be computationally infeasible to produce a valid signature for any message.
- Given the signatures produced by a signer, it shall be computationally infeasible to produce a valid signature on a new message or to recover the signature key.
- It shall be computationally infeasible, even for the signer, to find two different messages with the same signature.

NOTE Computational feasibility depends on the specific security requirements and environment.

Most digital signature mechanisms are based on asymmetric cryptographic techniques and involve three basic operations.

- A process for generating pairs of keys, where each pair consists of a private signature key and the corresponding public verification key.
- A process that uses the signature key, called the signature process.
- A process that uses the verification key, called the verification process.

There are two types of digital signature mechanisms.

- When, for a given signature key, two signatures produced for the same message are identical, the mechanism is said to be non-randomized (or deterministic); see ISO/IEC 14888-1.
- When, for a given message and signature key, each application of the signature process produces a different signature, the mechanism is said to be randomized.

The first and third of the three mechanisms specified in this part of ISO/IEC 9796 are deterministic (non-randomized), whereas the second of the three mechanisms specified is randomized.

Digital signature mechanisms can also be divided into the following two categories:

- When the whole message has to be stored and/or transmitted along with the signature, the mechanism is named a "signature mechanism with appendix" (see ISO/IEC 14888).
- When the whole message, or part of it, can be recovered from the signature, the mechanism is named a "signature mechanism giving message recovery" (see ISO/IEC 9796 (all parts)).

NOTE Any signature mechanism giving message recovery, for example, the mechanisms specified in ISO/IEC 9796 (all parts), can be converted to give a digital signature with appendix. This can be achieved by applying the signature mechanism to a hash-code derived as a function of the message. If this approach is employed, then all parties generating and verifying signatures must agree on this approach, and must also have a means of unambiguously identifying the hash-function to be used to generate the hash-code from the message.

The mechanisms specified in ISO/IEC 9796 (all parts) give either total or partial recovery, with the objective of reducing storage and transmission overhead. If the message is short enough, then the entire message can be included in the signature, and recovered from the signature in the verification process. Otherwise, a part of the message can be included in the signature, and the remainder stored and/or transmitted along with the signature.

The mechanisms specified in this part of ISO/IEC 9796 use a hash-function for hashing the entire message (possibly in more than one part). ISO/IEC 10118 specifies hash-functions for digital signatures.

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## Patent information

The International Organization for Standardization (ISO) and the International Electrotechnical Commission (IEC) draw attention to the fact that it is claimed that compliance with this part of ISO/IEC 9796 may involve the use of a patent concerning the "Probabilistic signature scheme" (U.S. Patent 6,266,771 issued 2001-07-24).

ISO and IEC take no position concerning the evidence, validity and scope of this patent right.

The holder of this patent right has assured ISO and IEC that they are willing to negotiate licences under reasonable and non-discriminatory terms and conditions with applications throughout the world. In this respect, the statement of the holder of this patent right is registered with ISO and IEC. Information may be obtained from:

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Attention is drawn to the possibility that some of the elements of this part of ISO/IEC 9796 may be the subject of patent rights other than that identified above. ISO and IEC shall not be held responsible for identifying any or all such patent rights.



# Information technology — Security techniques — Digital signature schemes giving message recovery —

## Part 2: Integer factorization based mechanisms

### 1 Scope

This part of ISO/IEC 9796 specifies three digital signature schemes giving message recovery, two of which are deterministic (non-randomized) and one of which is randomized. The security of all three schemes is based on the difficulty of factorizing large numbers. All three schemes can provide either total or partial message recovery.

The method for key production for the three signature schemes is specified in this part of ISO/IEC 9796. However, techniques for key management and for random number generation (as required for the randomized signature scheme), are outside the scope of this part of ISO/IEC 9796.

Users of this standard are, wherever possible, recommended to adopt the second mechanism (Digital signature scheme 2). However, in environments where generation of random variables by the signer is deemed infeasible, then Digital signature scheme 3 is recommended. Digital signature scheme 1 shall only be used in environments where compatibility is required with systems implementing the first edition of this standard. However, Digital signature scheme 1 is only compatible with systems implementing the first edition of this standard that use hash-codes of at least 160 bits.

### 2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO/IEC 9796. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO/IEC 9796 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO/IEC 9796-3:2000, *Information technology — Security techniques — Digital signature schemes giving message recovery — Part 3: Discrete logarithm based mechanisms*

ISO/IEC 9797-2, *Information technology — Security techniques — Message Authentication Codes (MACs) — Part 2: Mechanisms using a dedicated hash-function*

ISO/IEC 9798-1:1997, *Information technology — Security techniques — Entity authentication — Part 1: General*

ISO/IEC 10118 (all parts), *Information technology — Security techniques — Hash-functions*

ISO/IEC 14888 (all parts), *Information technology — Security techniques — Digital signatures with appendix*

### 3 Terms and definitions

For the purposes of this part of ISO 9796, the following terms and definitions apply.

#### 3.1

##### **capacity**

positive integer indicating the number of bits available within the signature for the recoverable part of the message.

**3.2**

**certificate domain**

collection of entities using public key certificates created by a single Certification Authority (CA) or a collection of CAs operating under a single security policy.

**3.3**

**certificate domain parameters**

cryptographic parameters specific to a certificate domain and which are known and agreed by all members of the certificate domain.

**3.4**

**collision-resistant hash-function**

hash-function satisfying the following property:

— it is computationally infeasible to find any two distinct inputs which map to the same output.

[ISO/IEC 10118-1: 2000]

**3.5**

**hash-code**

string of bits which is the output of a hash-function.

[ISO/IEC 10118-1: 2000]

**3.6**

**hash-function**

function which maps strings of bits to fixed-length strings of bits, satisfying the following two properties

— for a given output, it is computationally infeasible to find an input which maps to this output;

— for a given input, it is computationally infeasible to find a second input which maps to the same output.

[ISO/IEC 9797-2: 2001]

**3.7**

**mask generation function**

function which maps strings of bits to strings of bits of arbitrary specified length, satisfying the following property

— it is computationally infeasible to predict, given one part of an output but not the input, another part of the output.

**3.8**

**message**

string of bits of any length.

[ISO/IEC 14888-1: 1998]

**3.9**

**message representative**

bit string derived as a function of the message and which is combined with the private signature key to yield the signature.

**3.10**

**nibble**

block of four consecutive bits (half an octet).

**3.11**

**non-recoverable part**

part of the message stored or transmitted along with the signature; empty when message recovery is total.

**3.12**

**octet**

string of eight bits.

**3.13**

**private key**

that key of an entity's asymmetric key pair which should only be used by that entity.

[ISO/IEC 9798-1: 1997]

**3.14****private signature key**

private key which defines the private signature transformation.

[ISO/IEC 9798-1: 1997]

**3.15****public key**

that key of an entity's asymmetric key pair which can be made public.

[ISO/IEC 9798-1: 1997]

**3.16****public key system (for digital signature)**

cryptographic scheme consisting of three functions:

- *Key production*, a method for generating a key pair made up of a private signature key and a public verification key,
- *Signature production*, a method for generating a signature  $\Sigma$  from a message representative  $F$  and a private signature key, and
- *Signature opening*, a method for obtaining the recovered message representative  $F^*$  from a signature  $\Sigma$  and a public verification key. The output of this function also contains an indication as to whether the signature opening procedure succeeded or failed.

**3.17****public verification key**

public key which defines the public verification transformation.

[ISO/IEC 9798-1: 1997]

**3.18****recoverable part**

part of the message conveyed in the signature.

**3.19****salt**

random data item produced by the signing entity during the generation of the message representative in Signature scheme 2.

**3.20****signature**

string of bits resulting from the signature process.

[ISO/IEC 14888-1: 1998]

**3.21****trailer**

string of bits of length one or two octets, concatenated to the end of the recoverable part of the message during message representative production.

## 4 Symbols and abbreviated terms

For the purposes of this part of ISO/IEC 9796, the following symbols and abbreviations apply.

NOTE – In most cases upper case letters are used to represent bit strings and octet strings, whereas lower case letters are used to represent functions.

C Octet string encoding the bit length of the recoverable part of the message (used in message representative production in Signature schemes 2 and 3).

c The capacity of the signature scheme, i.e. the maximum number of bits available for the recoverable part of the message.

$c^*$  The recoverable message length, i.e. the length in bits of the recoverable part of the message ( $c \geq c^*$ ).

$D, D'$  Bit strings constructed during message representative production in Signature schemes 2 and 3.

$D^*, D'^*$  Bit strings constructed during message recovery in Signature schemes 2 and 3.

$F$  Message representative (a bit string).

$F^*$  Recovered message representative (as output from the Signature opening step).

$g$  Mask generation function.

$H$  Hash-code computed as a function of the message  $M$  (a bit string).

$H^*$  Recovered hash-code as derived during the Message recovery step.

$h$  Collision-resistant hash-function.

$k$  The bit length of the modulus of the private signature key and public verification key (see Annex A).

$L_h$  The bit length of hash-codes produced by the hash-function  $h$ .

$L_s$  The bit length of the salt  $S$ .

$M$  Message to be signed (a bit string).

$M^*$  Message recovered from a signature as a result of the verification process.

$M_1$  Recoverable part of the message  $M$ , i.e.  $M = M_1 || M_2$ .

$M_1^*$  Recovered recoverable part of the message (as generated during message recovery).

$M_2$  Non-recoverable part of the message  $M$ , i.e.  $M = M_1 || M_2$ .

$M_2^*$  Non-recoverable part of the message, as input to the verification process.

$N$  Bit string constructed during message representative production in Signature schemes 2 and 3.

$N^*$  Bit string generated during message recovery in Signature schemes 2 and 3.

$P$  A string of zero bits constructed during message representative production in Signature schemes 2 and 3.

$S$  Salt (a bit string).

$S^*$  Recovered salt (a bit string).

$t$  The number of octets in the Trailer field ( $t = 1$  or  $2$ ).

$T$  The Trailer field (a string of  $8t$  bits used during message representative production).

$\Delta$  Integer in the range 0 to 7 used in the specification of message allocation.

$\delta$  Integer in the range 0 to 7 used in the specification of Signature schemes 2 and 3.

$\Sigma$  Signature (a bit string containing  $k-1$  or  $k$  bits).

$|A|$  The bit length of the bit-string  $A$ , i.e. the number of bits in  $A$ .

$A || B$  Concatenation of bit strings  $A$  and  $B$  (in that order).

$\lceil a \rceil$  for a real number  $a$ , the smallest integer not less than  $a$ .

$a \bmod n$  for integers  $a$  and  $n$ ,  $(a \bmod n)$  denotes the (non-negative) remainder obtained when  $a$  is divided by  $n$ . Equivalently if  $b = a \bmod n$ , then  $b$  is the unique integer satisfying:

- (i)  $0 \leq b < n$ , and
- (ii)  $(b-a)$  is an integer multiple of  $n$ .

## 5 Converting between bit strings and integers

To represent a non-negative integer  $x$  as a bit string of length  $l$  ( $l$  has to be such that  $2^l > x$ ), the integer shall be written in its unique binary representation:

$$x = 2^{l-1}x_{l-1} + 2^{l-2}x_{l-2} + \dots + 2x_1 + x_0$$

where  $0 \leq x_i < 2$  (note that one or more leading digits will be zero if  $x < 2^{l-1}$ ). The bit string shall be

$$x_{l-1} x_{l-2} \dots x_0.$$

To represent a bit string  $x_{l-1} x_{l-2} \dots x_0$  (of length  $l$ ) as an integer  $x$ , the inverse process shall be followed, i.e.  $x$  shall be the integer defined by

$$x = 2^{l-1}x_{l-1} + 2^{l-2}x_{l-2} + \dots + 2x_1 + x_0.$$

## 6 Requirements

Users who wish to employ a digital signature mechanism compliant with this part of ISO/IEC 9796 shall ensure that the following properties hold.

- a) The message  $M$  to be signed shall be a binary string of any length, possibly empty.
- b) The signature function uses a private signature key, while the verification function uses the corresponding public verification key.
  - Each signing entity shall use and keep secret its private signature key corresponding to its public verification key.
  - Each verifying entity should know the public verification key of the signing entity.
- c) Use of the signature schemes specified in this standard requires the selection of a collision-resistant hash-function  $h$ . There shall be a binding between the signature mechanism and the hash-function in use. Without such a binding, an adversary might claim the use of a weak hash-function (and not the actual one) and thereby forge a signature.

NOTE 1 There are various ways to accomplish this binding. The following options are listed in order of increasing risk.

1. Require a particular hash-function when using a particular signature mechanism. The verification process shall exclusively use that particular hash-function. ISO/IEC 14888-3 gives an example of this option where the DSA mechanism requires the use of Dedicated Hash-function 3 from ISO/IEC 10118-3 (otherwise known as SHA-1).
2. Allow a set of hash-functions and explicitly indicate the hash-function in use in the certificate domain parameters. Inside the certificate domain, the verification process shall exclusively use the hash-function indicated in the certificate. Outside the certificate domain, there is a risk arising from certification authorities (CAs) that may not adhere to the user's policy. If, for example, an external CA creates a certificate permitting other hash-functions, then signature forgery problems may arise. In such a case a misled verifier may be in dispute with the CA that produced the other certificate.
3. Allow a set of hash-functions and indicate the hash-function in use by some other method, e.g., an indication in the message or a bilateral agreement. The verification process shall exclusively use the hash-function indicated by the other method. However, there is a risk that an adversary may forge a signature using another hash-function.

NOTE 2 The 'other method' referred to in paragraph 3 immediately above could be in the form of a hash-function identifier included in the message representative  $F$  (see clauses 8.1.2 and 9.1.3). If the hash-function identifier is included in  $F$  in this way then an attacker cannot fraudulently reuse an existing signature with the same  $M_1$  and a different  $M_2$ , even when

the verifier could be persuaded to accept signatures created using a hash-function sufficiently weak that pre-images can be found. However, as discussed in detail in [11] (see also Annex C), in this latter case and using the weak hash-function, an attacker can still find a new signature with a 'random'  $M_1$ .

NOTE 3 The attack mentioned in Note 2 that yields a new signature with a 'random'  $M_1$  can be prevented by requiring the presence of a specific structure in  $M_1$ . For instance, one may impose a length limit on  $M_1$  that is sufficiently less than the capacity of the signature scheme (see Annex C for further discussion). For digital signature schemes 2 and 3, a length limit on  $M_1$  may also prevent an attacker from reusing existing signatures even if no hash-function identifier is included in the message representative, provided that the mask generation function  $g$  is based on the hash-function. This holds under the reasonable assumption that the weak hash-function involved is a 'general purpose' hash-function, not one designed solely for the purpose of forging a signature.

The user of a digital signature mechanism should conduct a risk assessment considering the costs and benefits of the various alternative means of accomplishing the required binding. This assessment should include an assessment of the cost associated with the possibility of a bogus signature being produced.

- d) The verifier of a signature shall always have a secure independent means of determining which of the three signature schemes specified in this standard have been employed to generate the signature. In addition, if Digital signature scheme 2 or 3 is being used, the signature verifier shall also have a means of determining which of the two signature production functions specified in Annex A have been used. This could, for example, be achieved by specifying the mechanism and signature production function in agreed 'domain parameters' or by including an unambiguous identifier for the signature scheme and signature production function in the signer's public key certificate. The signature production function may also be specified in an algorithm identifier associated with the signed data.
- e) The digital signature schemes specified in this part of ISO/IEC 9796 each have particular options, the range of possible choices of which by the signer must be known to the verifier by a secure independent means. These options are as follows.
  - For all three digital signature schemes, the verifier must know whether trailer field option 1 or 2 is being employed.
  - For digital signature schemes 2 and 3, the verifier must know  $L_S$ , the length of the salt  $S$ .

This could, for example, be achieved by specifying the option selection in the 'domain parameters' or by including option information in the signer's public key certificate.

## 7 Model for signature and verification processes

The model for a signature scheme giving message recovery presented here applies to all three of the schemes in this part of ISO/IEC 9796. When applied to a message  $M$ , a signature scheme of this type can provide either total or partial message recovery.

- If  $M$  is sufficiently short, then message recovery can be total because it is possible for  $M$  to be entirely included in the signature.
- If  $M$  is too long, then message recovery will be partial. In this case  $M$  shall be divided into the *recoverable part*, a string of bits of limited length to be included in the signature, and the *non-recoverable part*, a string of octets of any length to be stored and/or transmitted along with the signature.

The model is divided into three parts: a specification of the procedure for signing a message, a specification of the procedure for verifying a signature, and details of the additional aspects of signing and verifying that need to be defined in order to complete the specification of a signature scheme. Clauses 8, 9 and 10 specify these additional aspects for the three schemes defined in this part of ISO/IEC 9796.

## 7.1 Signing a message

### 7.1.1 Overview

Given a message  $M$  to be signed, three steps need to be performed to generate a signature on  $M$ , namely message allocation, recoverable string production, and signature production.

- *Message allocation* consists of the process whereby the message is divided into two parts: a recoverable part  $M_1$  and a non-recoverable part  $M_2$  (which may be empty). The length of the recoverable part is bounded above by the capacity  $c$  of the signature scheme, a value determined by the choice of the signature scheme and the key for the scheme. The recoverable part will be recovered from the signature during the verification process, whereas the non-recoverable part must be made available to the verifier by other means (e.g. it can be sent or stored with the signature). Hence, if the message is sufficiently short, the entire message can be allocated to the recoverable part, and the non-recoverable part will be empty.
- *Message representative production* takes as input the two parts of the message, and outputs a formatted string, known as the *message representative*, which is input to the signature production step.
- *Signature production* takes as input the message representative and the private signature key and outputs the *signature*  $\Sigma$ . This process is performed using a public key system.

### 7.1.2 Message allocation

The choice of signature scheme and key for the scheme determine the capacity  $c$  of the signature, where  $c$  must satisfy  $c \geq 7$ . The message  $M$  to be signed shall be divided into two parts,  $M_1$  and  $M_2$ , as follows.

A recoverable message length  $c^*$  shall be chosen, where  $c^* \leq c$ ,  $c^* \leq |M|$ , and  $c^* \equiv |M| \pmod{8}$ . For Signature scheme 1,  $c^*$  shall be set equal to the minimum of  $c-\Delta$  and  $|M|$ , where  $\Delta = (c-|M|) \bmod 8$ .

- If  $|M| = c^*$  then the entire message shall be recoverable, i.e.  $M_1 = M$  and  $M_2$  shall be empty.
- If  $|M| > c^*$  then  $M_1$  shall be set equal to the left-most  $c^*$  bits of  $M$ , and  $M_2$  shall be set equal to the remainder of  $M$ , i.e.  $M_2$  contains  $|M|-c^*$  bits.

In either case it follows that  $M = M_1||M_2$ .

NOTE 1 For practical purposes, an application may wish to structure the message  $M$  to ensure that data it wants to be explicitly stored or transmitted (e.g. address information) is allocated to the non-recoverable message part  $M_2$ . However, the structure and interpretation of the message  $M$  are outside the scope of this part of ISO/IEC 9796.

NOTE 2 The method for message allocation ensures that  $M_2$  is always a whole number of octets in length. Moreover, choosing  $c^*$  to be the minimum of  $c-\Delta$  and  $|M|$ , where  $\Delta = (c-|M|) \bmod 8$ , ensures that  $M_1$  is as long as possible subject to this constraint. Also, if  $M$  is a whole number of octets in length, i.e. if  $|M|$  is an integer multiple of 8, then both  $M_1$  and  $M_2$  will consist of a whole number of octets.

### 7.1.3 Message representative production

This step takes as input the recoverable and non-recoverable parts of the message,  $M_1$  and  $M_2$ , and outputs the message representative  $F$ . This shall be achieved using one of the methods specified in clauses 8, 9 and 10 of this part of ISO/IEC 9796. These methods require use of a hash-function  $h$  and, in the cases of the second and third mechanisms, a mask generation function  $g$  that also uses  $h$ . The hash-function  $h$  to be used shall be selected from amongst those standardised in ISO/IEC 10118; the mask generation function  $g$  shall be set equal to the function specified in Annex B of this part of ISO/IEC 9796.

### 7.1.4 Signature production

This step takes as input the message representative  $F$  and the private signature key and outputs the *signature*  $\Sigma$ . This shall be achieved using the public key system specified in Annex A to this part of ISO/IEC 9796.

## 7.2 Verifying a signature

### 7.2.1 Overview

A signed message consists of either the signature  $\Sigma$  alone in the case of total recovery, or the non-recoverable part of the message  $M_2^*$  together with the signature  $\Sigma$  in the case of partial recovery. A signature shall be accepted if and only if the verification process is successful.

Given a signature  $\Sigma$  and non-recoverable message part  $M_2^*$ , three steps need to be performed to verify  $\Sigma$  and recover  $M^*$ , namely signature opening, message recovery and message assembly.

- *Signature opening* takes as input the signature  $\Sigma$  and the public verification key and outputs a recovered message representative  $F^*$  or returns an indication that verification has failed. This process is performed using a public key system.
- *Message recovery* takes as input the recovered message representative  $F^*$  and the non-recoverable part of the message  $M_2^*$ , and outputs the (recovered) recoverable part of the message  $M_1^*$ , or returns an indication that verification has failed.
- *Message assembly* consists of the process whereby the recovered message  $M^*$  is reconstituted from the (recovered) recoverable part  $M_1^*$  and the non-recoverable part  $M_2^*$  (which may be empty).

### 7.2.2 Signature opening

This step takes as input the signature  $\Sigma$  and the public verification key and either outputs a recovered message representative  $F^*$  or returns an indication that verification has failed. This shall be achieved using the public key system specified in Annex A to this part of ISO/IEC 9796.

### 7.2.3 Message recovery

This step takes as input the recovered message representative  $F^*$  and the non-recoverable part of the message  $M_2^*$ , and outputs the recoverable part of the message  $M_1^*$ , or returns an indication that verification has failed. This shall be achieved using one of the methods specified in clauses 8, 9 and 10 of this part of ISO/IEC 9796. These methods require use of a hash-function and, in the case of the second and third mechanisms, a mask generation function. The hash-function to be used shall be selected from amongst those standardised in ISO/IEC 10118; the mask generation function shall be set equal to the function specified in Annex B of this part of ISO/IEC 9796.

### 7.2.4 Message assembly

This step consists of the process whereby the message  $M^*$  is reconstituted from the recoverable part  $M_1^*$  and the non-recoverable part  $M_2^*$  (which may be empty). That is, the message  $M^*$  is recovered as  $M^* = M_1^*||M_2^*$ .

## 7.3 Specifying a signature scheme

The purpose of this clause is to define what choices need to be made to uniquely specify the signing and verification processes specified in this part of ISO/IEC 9796.

- a) The message allocation and message assembly steps are uniquely defined within this part of ISO/IEC 9796.
- b) One of the three options for the message representative production and message recovery steps, as defined in Clauses 8, 9 and 10 of this part of ISO/IEC 9796, must be chosen. Whichever of these three options is selected, a hash-function must also be chosen, which shall be selected from amongst those standardised in ISO/IEC 10118 subject to the constraint that the hash-code output shall contain at least 160 bits. In two of the three cases a mask generation function is additionally required, and the function to be employed is defined in Annex B of this part of ISO/IEC 9796.
- c) The signature production and signature opening steps are uniquely defined within Annex A of this part of ISO/IEC 9796, up to the choice of the private signature key used in the signature production process and, in the case of Signature schemes 2 and 3 with an odd exponent, up to the choice between the basic and

alternative signature and verification functions. The method to be used to generate pairs of private signature keys and public verification keys is defined in Annex A of this part of ISO/IEC 9796.

## 8 Digital signature scheme 1

This clause defines the message representative production and message recovery processes for a deterministic digital signature scheme giving message recovery.

### 8.1 Parameters

#### 8.1.1 Modulus length

The private signature key in use is assumed to have a modulus of length  $k$  bits (see Annex A). This determines both  $c$ , the capacity of the signature, and the length of  $F$ , the message representative.

#### 8.1.2 Trailer field options

In this scheme the trailer field (used as part of the construction of the Message representative) may be either one or two octets in length. The trailer shall consist of  $t$  octets ( $t = 1$  or  $2$ ) where the rightmost nibble shall always be equal to hexadecimal 'C'. The following two options are permitted.

- Option 1 ( $t = 1$ ): the trailer shall consist of a single octet; this octet shall be equal to hexadecimal 'BC'.
- Option 2 ( $t = 2$ ): the trailer shall consist of two consecutive octets; the rightmost octet shall be equal to hexadecimal 'CC' and the leftmost octet shall be the hash-function identifier. The hash-function identifier indicates the hash-function in use.

The range '00' to '7F' is reserved for ISO/IEC JTC1; ISO/IEC 10118 specifies a unique identifier in that range for every standardized hash-function. The range '80' to 'FF' is reserved for proprietary use.

NOTE Use of the second option does not avoid the need for the verifier to have a secure independent means of knowing which hash-function to use to verify the signature. Whilst this was previously believed to be the case, this has been shown to be false, [11] (see also Annex C).

#### 8.1.3 Capacity

The capacity  $c$  of the signature for this scheme is given by:

$$c = k - L_h - 8t - 4.$$

## 8.2 Message representative production

In this scheme message representative production involves two main steps:

- hashing the message;
- formatting.

### 8.2.1 Hashing the message

The message  $M$  (where  $M = M_1||M_2$ ) shall be input to the hash-function  $h$  to obtain the hash-code  $H$ , i.e.,  $H = h(M)$ . Note that  $H$  contains  $L_h$  bits.

### 8.2.2 Formatting

A string of  $k$  bits shall be constructed as follows (working from left to right):

- two bits set equal to '01'
- a single bit set equal to '0' in the case of total recovery (i.e. when  $M = M_1$ ) and '1' in the case of partial recovery (i.e. when  $|M_2| > 0$ ),
- $k - L_h - |M_1| - 8t - 4$  padding bits all set to '0',
- a single bit set equal to '1' (the final padding bit),
- the  $|M_1|$  bits of  $M_1$ ,
- the  $L_h$  bits of  $H$ , the hash-code,
- the  $8t$  bits of the trailer field  $T$ .

NOTE Where partial recovery is provided,  $M_2$  is kept as short as possible subject to the constraint that it shall be a whole number of octets; in this case the number of padding bits equal to '0' will be less than eight.

The message representative  $F$  shall result from processing the above string from left to right in blocks of four consecutive bits, i.e., in nibbles, following the steps below.

1. The leftmost nibble shall remain unchanged.
2. If the leftmost nibble has its rightmost bit set to '0' then
  - a) every subsequent nibble equal to '0000', if any, shall be replaced by a nibble equal to hexadecimal 'B'; it is part of the padding field.
  - b) the first subsequent nibble not equal to '0000' shall be exclusive-ored with hexadecimal 'B' (i.e., '1011'); this is the nibble containing the final padding bit.
3. All subsequent bits shall remain unchanged.

NOTE This means that if the left-most nibble has its rightmost bit set to '1' (and hence there are no '0' padding bits), then no changes are made to the bit string.

4. The first bit of the resulting string (which will always be equal to '0') shall be deleted, resulting in  $F$ , a string of  $k-1$  bits.

### 8.3 Message recovery

As specified in clause 6, the verifier must know which hash-function  $h$  is in use prior to processing a signature. Hence the verifier will also know  $L_h$ .

If the rightmost octet of the recovered message representative  $F^*$ , a string of  $k-1$  bits, is equal to

- hexadecimal 'BC', then the trailer consists of that single octet;
- hexadecimal 'CC', then the trailer consists of the rightmost two octets of  $F^*$ , where the leftmost octet is the identifier of the hash-function in use. This should be checked to determine whether it equals the hash-function in use by the verifier; if it disagrees then the signature verification has failed.

The signature  $\Sigma$  shall be rejected if either the trailer or the hash-function identifier (if present) cannot be interpreted. Otherwise, the verification process shall continue.

The signature  $\Sigma$  shall be rejected if the leftmost bit of the recovered message representative  $F^*$  is '0'.

A single '0' bit shall be adjoined at the left end of the string (resulting in a string of  $k$  bits). This string shall next be processed from left to right in blocks of four consecutive bits, i.e., in nibbles, following the steps below.

1. The leftmost nibble shall remain unchanged.
2. If the leftmost nibble has its rightmost bit set to '0' then
  - a) every subsequent nibble equal to hexadecimal 'B', if any, is part of the padding field,
  - b) the first subsequent nibble not equal to hexadecimal 'B' shall be exclusive-ored with hexadecimal 'B' to recover the initial value of this nibble.
3. All subsequent bits shall remain unchanged.

The location of the final (rightmost) padding bit can now be determined, and hence the total number of padding bits calculated. The third bit of the first nibble can also be processed to determine whether the signature provides partial or total recovery. In the case of partial recovery, the signature  $\Sigma$  shall be rejected if nine or more padding bits are present (i.e. eight or more zero padding bits). Otherwise, the verification process shall continue.

All bits up to the end of the padding field shall be removed from the left of the modified version of  $F^*$ , and the one or two-octet trailer shall be removed from the right. The remaining binary string shall be divided into two parts.

- The recovered hash-code  $H^*$  shall consist of the rightmost  $L_h$  bits.
- The recovered part of the message  $M_1^*$  shall consist of the remaining bits on the left.

The recovered message part  $M_1^*$  shall be concatenated with  $M_2$ , the non-recoverable part of the message, as submitted to the verification process, and submitted to the hash-function. If the result is the same as  $H^*$ , i.e. if  $H^* = h(M_1^*||M_2)$ , then the signature shall be accepted and  $M_1^*$  shall be returned; otherwise the signature shall be rejected.

## 9 Digital signature scheme 2

This clause defines the message representative production and message recovery processes for a randomized digital signature scheme giving message recovery.

**NOTE** This signature scheme is compatible with the scheme known as IFSSR specified in IEEE P1363a, [9]. It is closely based on a scheme known as PSS-R, [3]. The message representative production method is similarly derived from the method known as EMSR3 in IEEE P1363a, [9].

### 9.1 Parameters

#### 9.1.1 Modulus length

The private signature key in use is assumed to have a modulus of length  $k$  bits (see Annex A). This determines both  $c$ , the capacity of the signature, and the length of  $F$ , the message representative.

#### 9.1.2 Salt length

A salt length  $L_S$  shall be selected.  $L_S$  shall be a positive integer ( $L_S > 0$ ); a typical value is  $L_h$ .

#### 9.1.3 Trailer field options

In this scheme the trailer field (used as part of the construction of the message representative) may be either one or two octets in length. The trailer shall consist of  $t$  octets ( $t = 1$  or  $2$ ) where the rightmost nibble shall always be equal to hexadecimal 'C'. The following two options are permitted.

- Option 1 ( $t = 1$ ): the trailer shall consist of a single octet; this octet shall be equal to hexadecimal 'BC'.

- Option 2 ( $t = 2$ ): the trailer shall consist of two consecutive octets; the rightmost octet shall be equal to hexadecimal ‘CC’ and the leftmost octet shall be the hash-function identifier. The hash-function identifier indicates the hash-function in use.

The range ‘00’ to ‘7F’ is reserved for ISO/IEC JTC1; ISO/IEC 10118 specifies a unique identifier in that range for every standardized hash-function. The range ‘80’ to ‘FF’ is reserved for proprietary use.

#### 9.1.4 Capacity

The capacity  $c$  of the signature for this scheme is given by:

$$c = k - L_h - L_S - 8t - 2.$$

### 9.2 Message representative production

In this scheme message representative production involves two main steps:

- hashing the message;
- formatting.

#### 9.2.1 Hashing the message

The hash-code  $H$  shall be computed using the following or an equivalent sequence of steps.

1. Convert the bit length of  $M_1$ , i.e.  $|M_1|$ , to a bit string  $C$  of length 64 bits using the convention described in clause 5.
2. Generate a fresh, random, bit string  $S$  of length  $L_S$  bits.
3. Compute the hash-code  $H$  as  $H = h(C \parallel M_1 \parallel h(M_2) \parallel S)$ . Note that  $H$  contains  $L_h$  bits.

#### 9.2.2 Formatting

The message representative  $F$  shall be computed by the following or an equivalent sequence of steps.

1. Let  $P$  be the bit string that consists of  $k + \delta - L_h - L_S - |M_1| - 8t - 2$  ‘0’ bits where  $\delta = (1-k) \bmod 8$ .
2. Let the bit string  $D$  be defined by  $D = P \parallel '1' \parallel M_1 \parallel S$ , where ‘1’ is a single bit. The length of  $D$  is  $k + \delta - L_h - 8t - 1$  bits.
3. Apply the mask generation function  $g$  to the hash-code  $H$  to produce a bit string  $N$  of length  $k + \delta - L_h - 8t - 1$  bits.
4. The length of  $D \oplus N$  is  $k + \delta - L_h - 8t - 1$  bits. Let  $D'$  be the string of  $k - L_h - 8t - 1$  bits obtained by deleting the leftmost  $\delta$  bits of  $D \oplus N$ .
5. Let  $F = D' \parallel H \parallel T$ , where  $T$  is the Trailer field of  $8t$  bits.  $F$  is a string of  $k-1$  bits.

### 9.3 Message recovery

If the rightmost octet of the recovered message representative  $F^*$ , a string of  $k-1$  bits, is equal to

- hexadecimal ‘BC’, then the trailer consists of that single octet;

- hexadecimal ‘CC’, then the trailer consists of the rightmost two octets of  $F^*$ , where the leftmost octet is the identifier of the hash-function in use. This should be checked to determine whether it equals the hash-function in use by the verifier; if it disagrees then the signature verification has failed.

The signature  $\Sigma$  shall be rejected if either the trailer or the hash-function identifier (if present) cannot be interpreted. Otherwise, the verification process shall continue.

The recoverable message part  $M_1$  shall next be recovered from the recovered message representative  $F^*$  and the non-recoverable part  $M_2$  by the following or an equivalent sequence of steps.

1. Adjoin  $\delta$  ‘0’ bits to the leftmost end of  $F^*$ .
2. Let  $D'^*$  be the leftmost  $k + \delta - L_h - 8t - 1$  bits of the resulting string, and  $H^*$  be the next  $L_h$  bits.
3. Apply the mask generation function  $g$  to the string  $H^*$  to produce a bit string  $N^*$  of length  $k + \delta - L_h - 8t - 1$  bits.
4. Let  $D^* = D'^* \oplus N^*$ .
5. Set the leftmost  $\delta$  bits of  $D^*$  to ‘0’.
6. Working from the leftmost end of  $D^*$ , search for the first ‘1’ bit. Remove this bit and all zeros to the left of this bit, and then let  $S^*$  be the rightmost  $L_s$  bits of  $D^*$ , and  $M_1^*$  be the remaining bits of  $D^*$ . If there is no first ‘1’ bit, return an indication that verification has failed and stop.
7. Convert the bit length of  $M_1^*$  to a bit string  $C$  of length 64 bits using the convention described in clause 5.
8. If  $H^* = h(C \parallel M_1^* \parallel h(M_2) \parallel S^*)$  output the recovered message part  $M_1^*$ . Otherwise, return an indication that verification has failed.

## 10 Digital signature scheme 3

This clause defines the message representative production and message recovery processes for a deterministic digital signature scheme giving message recovery.

This scheme is identical to that defined in clause 9 with the exception that  $S$  is a fixed value which is permitted to have zero length, i.e.  $L_s \geq 0$  (unlike the constraint  $L_s > 0$  which applies in Clause 9). Hence this scheme is deterministic and not randomized.

The fixed salt  $S$  may be selected by the signer. Alternatively, it may be specified as part of the domain parameters.

NOTE 1 The security of this scheme is of a similar level to that obtainable from the use of ‘full domain hashing’, [1], [5].

NOTE 2 Digital signature scheme 3 is deemed to be preferable to Digital signature scheme 1 – see clause 1. This is for the following reasons.

- Schemes very similar to Digital signature scheme 3 have mathematical proofs of security (see [5]). However, these proof techniques do not apply to Digital signature scheme 1.
- The two schemes have comparable efficiencies.

## Annex A (normative)

### Public key system for digital signature

In this annex a public key system is defined. This public key system has three main parts:

- *Key production*, a method for generating a key pair made up of a private signature key and a public verification key,
- *Signature production*, a method for generating a signature  $\Sigma$  from a message representative  $F$  and a private signature key, and
- *Signature opening*, a method for obtaining the recovered message representative  $F^*$  from a signature  $\Sigma$  and a public verification key. The output of this function also contains an indication as to whether the signature opening procedure succeeded or failed.

#### A.1 Terms and definitions

For the purposes of this annex, the following terms and definitions apply.

##### A.1.1

##### modulus

integer equal to the product of two primes, and which constitutes part of the public and private keys.

##### A.1.2

##### private signature key

modulus and private signature exponent.

##### A.1.3

##### public verification key

modulus and public verification exponent.

#### A.2 Symbols and abbreviations

For the purposes of this annex, and in addition to the symbols defined in clause 4, the following symbols and abbreviations apply.

|        |   |
|--------|---|
| $I$    | the integer having $F$ as its binary representation                             |
| $I^*$  | an integer calculated during signature opening                                  |
| $J$    | an integer calculated during signature production                               |
| $J^*$  | an integer calculated during signature opening                                  |
| $n$    | the modulus (part of the private signature key and the public verification key) |
| $p, q$ | prime factors of the modulus  |
| $s$    | signature exponent  |
| $v$    | verification exponent   |

$\text{lcm}(a, b)$  the least common multiple of integers  $a$  and  $b$

$\min\{a, b\}$  the smaller of the two values  $a$  and  $b$ .

$(a | n)$  Jacobi symbol of  $a$  with respect to  $n$

NOTE 1 Let  $p$  be an odd prime, and let  $a$  be a positive integer. The following formula defines the Legendre symbol of  $a$  with respect to  $p$ .

$$(a | p) = a^{(p-1)/2} \bmod p$$

The Legendre symbol of multiples of  $p$  with respect to  $p$  is 0. When  $a$  is not a multiple of  $p$ , the Legendre symbol of  $a$  with respect to  $p$  is either +1 or -1 depending on whether  $a$  is or is not a square modulo  $p$ .

NOTE 2 Let  $n$  be an odd positive integer, and let  $a$  be a positive integer. The Jacobi symbol of  $a$  with respect to  $n$  is the product of the Legendre symbols of  $a$  with respect to the prime factors of  $n$  (repeating the Legendre symbols for repeated prime factors). Therefore if  $n = p q$ , then  $(a | n) = (a | p) (a | q)$ . The Jacobi symbol of  $a$  with respect to  $n$  may be efficiently computed without knowledge of the prime factors of  $n$ .

## A.3 Key production

NOTE No method is specified in this document for public-key validation, that is providing assurance to a party (that is, the party generating the key pair, the party using the public key, or a neutral third party) that a candidate public key conforms to the arithmetic definition of a public key. An invalid public key might exist because of an inadvertent key generation calculation error or the deliberate action of an adversary. Use of an invalid public key should be assumed to void all security assurances, including the inability of an adversary to forge a signature or discover the associated private key. Users that desire assurance of the arithmetic validity of a public key before using it should use other methods, such as those in ISO/IEC 9796-3.

As a general principle for any cryptosystem, the use of an improperly generated but otherwise valid public key (for instance, one produced from an insufficiently random source), or an improperly protected private key, may also void all security assurances. Implementation validation can mitigate these risks as well as the possibility that invalid keys are used. However, it does not provide specific assurance that a given public key is in fact valid.

### A.3.1 Public verification exponent

Each signing entity shall select a positive integer  $v$  as its public verification exponent. The public verification exponent may be standardized in specific applications.

NOTE The values 2, 3, 17 and 65537 may have some practical advantages.

### A.3.2 Secret prime factors and public modulus

Each signing entity shall secretly and randomly select two distinct large primes  $p$  and  $q$  subject to the following conditions.

- If  $v$  is odd, then  $p-1$  and  $q-1$  shall be coprime to  $v$ .
- If  $v$  is even, then  $(p-1)/2$  and  $(q-1)/2$  shall be coprime to  $v$ . Moreover,  $p$  and  $q$  shall not be congruent to each other modulo 8.

The public modulus  $n$  is set equal to the product of  $p$  and  $q$ , i.e.  $n = pq$ . The size of modulus  $n$  in bits determines the value of  $k$  such that

$$2^{k-1} < n \leq 2^k - 1.$$

NOTE 1 Some additional conditions on the choice of primes may be taken into account in order to prevent factorization of the modulus.

NOTE 2 Some forms of the modulus simplify the modular reduction and need less table storage. Examples of such forms are

$$n = 2^{64x} - r \text{ of length: } k = 64x \text{ bits,}$$

$n = 2^{64x} + r$  of length:  $k = 64x + 1$  bits,

where:  $1 \leq y \leq 2x$  and  $r < 2^{64x-8y} < 2r$ .

For moduli of the form  $2^{64x} - r$ , the most significant  $8y$  bits are equal to 1, where  $8y$  is at most one quarter of the modulus length. For moduli of the form  $2^{64x} + r$ , the most significant bit is equal to 1 and is followed by  $8y$  bits equal to 0, where  $8y$  is at most one quarter of the modulus length.

### A.3.3 Private signature exponent

The private signature exponent shall be any positive integer  $s$  such that  $sv - 1$  is a multiple of

- $\text{lcm}(p-1, q-1)$  if  $v$  is odd;
- $\text{lcm}(p-1, q-1)/2$  if  $v$  is even.

NOTE Generally,  $s$  is the least possible value.

### A.4 Signature production function

The message representative  $F$  is a string of  $k-1$  bits, where the rightmost four bits are equal to '1100' (hexadecimal 'C'). It is the binary representation of a positive integer denoted by  $I$ .

The integer  $J$  is then defined as follows:

- if  $v$  is odd, then  $J = I$ ,
- if  $v$  is even and  $(I \mid n) = +1$ , then  $J = I$ , and
- if  $v$  is even and  $(I \mid n) = -1$ , then  $J = I/2$ .

NOTE If  $v$  is even, then the above operation ensures that the Jacobi symbol of  $J$  with respect to  $n$  is always +1.

The signature  $\Sigma$  is the bit string of length  $k-1$  bits corresponding to the integer  $\min\{J^s \bmod n, n - (J^s \bmod n)\}$  using the convention described in Clause 5.

### A.5 Signature opening function

The signature  $\Sigma$  is a string of  $k-1$  bits; it is the binary representation of a positive integer less than  $n$ . This integer shall be raised to the power  $v$  modulo  $n$  to get  $J^*$ , i.e.,

$$J^* = \Sigma^v \bmod n$$

The integer  $I^*$  shall then be computed as follows.

- If  $v$  is odd and
  - if  $J^* \bmod 16 = 12$ , then  $I^* = J^*$ ,
  - if  $J^* \bmod 16 = n-12 \bmod 16$ , then  $I^* = n-J^*$ .
- If  $v$  is even and
  - if  $J^* \bmod 8 = 1$ , then  $I^* = n-J^*$ ,
  - if  $J^* \bmod 8 = 4$ , then  $I^* = J^*$ ,

- if  $J^* \bmod 8 = 6$ , then  $I^* = 2J^*$ ,
- if  $J^* \bmod 8 = 7$ , then  $I^* = 2(n-J^*)$ ,

The signature  $\Sigma$  shall be rejected in all the other cases; it shall also be rejected if  $I^* \bmod 16 \neq 12$ , and if  $I^*$  does not satisfy  $I^* \leq 2^{k-1}-1$ .

The recovered message representative  $F^*$  is the bit string of length  $k-1$  bits corresponding to the integer  $I^*$  using the convention described in Clause 5.

## A.6 Alternative signature production function

When  $v$  is odd this function may be used as an alternative to the function in clause A.4. It shall be used together with the signature opening function in A.7.

The message representative  $F$  is a string of  $k-1$  bits, where the rightmost four bits are equal to '1100' (hexadecimal 'C'). It is the binary representation of a positive integer denoted by  $I$ .

The integer  $J$  is then defined as  $J = I$ .

The signature  $\Sigma$  is the bit string of length  $k$  bits corresponding to the integer  $J^s \bmod n$  using the convention described in Clause 5.

**NOTE** The difference between this function and the one in clause A.4 is that the signature  $S$  is always  $J^s \bmod n$ ; no "absolute value" step is performed to select the minimum of  $J^s \bmod n$  and  $n-(J^s \bmod n)$ .

## A.7 Alternative signature opening function

When  $v$  is odd this function may be used as an alternative to the function in clause A.5. It shall be used together with the signature production function in A.6.

The signature  $\Sigma$  is a string of  $k$  bits; it is the binary representation of a positive integer less than  $n$ . This integer shall be raised to the power  $v$  modulo  $n$  to get  $J^*$ , i.e.,

$$J^* = S^v \bmod n.$$

The integer  $I^*$  shall then be computed as  $I^* = J^*$ .

The signature  $\Sigma$  shall be rejected if  $I^* \bmod 16 \neq 12$ , and if  $I^*$  does not satisfy  $I^* \leq 2^{k-1}-1$ .

The recovered message representative  $F^*$  is the bit string of length  $k-1$  bits corresponding to the integer  $I^*$  using the convention described in Clause 5.

**NOTE** The difference between this function and the one in clause A.5 is that the integer  $I^*$  is always the same as the integer  $J^*$ , no "disambiguation" between  $J^*$  and  $n-J^*$  is necessary.

## Annex B (normative)

### Mask generation function

In this annex a mask generation function based on a hash-function is defined.

NOTE This function extends the one defined as MGF1 in IEEE Std 1363-2000, [8], to allow the input and output to be bit strings. It is similar to the proposals made by Bellare and Rogaway in [2] and [3].

A mask generation function takes as input a bit string  $Z$  and the desired length of the output,  $L_N$ , and outputs a bit string  $N$  of that length.

#### B.1 Symbols and abbreviations

For the purposes of this annex, and in addition to the symbols defined in clause 4, the following symbols and abbreviations apply.

$L_N$  The length (in bits) of the output from the mask generation function  $g$ .

$L_Z$  The length (in bits) of the octet string  $Z$ .

$N$  The output of the mask generation function  $g$  (a bit string).

$Z$  A bit string input to the mask generating function  $g$ .

#### B.2 Requirements

Use of this function requires the choice of a hash-function. This hash-function, denoted here by  $h$ , shall be set equal to the hash-function  $h$  as in clause 6, paragraph (c). We denote the output length of  $h$  in bits by  $L_h$ .

#### B.3 Specification

##### B.3.1 Parameters

One input to the function  $g$  is the desired length in bits of the output, which is a positive integer  $L_N$ .

##### B.3.2 Mask generation

The bit string  $N$  shall be computed by the following or an equivalent sequence of steps.

1. If  $L_Z$  exceeds the length limitation ( $2^{64} - 33$  for Dedicated Hash-functions 1 and 3 from ISO/IEC 10118-3), or if  $L_N > L_h \times 2^{32}$ , output “error” and stop.
2. Let  $N$  be the empty string.
3. Let  $i = 0$ .
  - 3.1 Convert  $i$  to a bit string  $C$  of length 32 bits using the convention described in clause 5.
  - 3.2 Let  $N := N \parallel h(Z \parallel C)$ .

- 3.3 Let  $i := i+1$ .
- 3.4 If  $i < \lceil L_N/L_h \rceil$ , go to Step 3.1.
4. Output the leftmost  $L_N$  bits of  $N$ .

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## Annex C

(informative)

### On hash-function identifiers and the choice of the recoverable length of the message

As specified in clause 6 (Requirements), users of signature schemes specified in this standard must select a collision-resistant hash-function  $h$ . It is important that the verifier has an unambiguous way of determining which hash-function was used to generate a signature, in order that the verification process can be carried out securely. If a malicious third party could persuade a verifier that a 'weak' hash-function had been used to generate a signature (e.g. a hash-function which lacks the one-way property), then this third party could persuade the verifier that a valid signature actually applies to a 'false' message.

The three digital signature schemes specified in this part of ISO/IEC 9796 allow a hash-function identifier to be included in the message representative  $F$  (see clause 8.1.2). If the hash-function identifier is included in  $F$  in this way then an attacker cannot fraudulently reuse an existing signature with the same  $M_1$  and a different  $M_2$ , even when the verifier could be persuaded to accept signatures created using a hash-function sufficiently weak that pre-images can be found. This was thought to solve the problem referred to in the previous paragraph.

However, as discussed in detail in [11], even if a hash-function identifier is included in the message representative, other attacks are possible if a verifier can be persuaded that a 'weak' hash-function has been employed. By weak here we mean a hash-function which lacks the one way property, i.e. given a hash-code it is computationally feasible to find an input string which is mapped to this hash-code by the hash-function. (Note that it is precisely this type of weakness that first motivated the inclusion of a hash-function identifier in the message representative).

The attacks described in [11] operate in the following general way. The attacker generates 'signatures' at random, and for each such 'signature' applies the public verification function of the entity whose signature he wishes to forge, and obtains the 'recovered message representative' (this is the 'signature opening' step). The next part of the attack will vary depending on the formatting of the message representative, but essentially the attacker sees if the recovered message representative is in the correct format to correspond to a genuine signature and that the hash-function identifier in this string is the one corresponding to a weak hash-function. The odds of this happening will vary, but could be as high as  $2^{-16}$  (and hence the attacker does not need to try too many 'random signatures' before finding one with the desired properties).

Given such a 'signature', the attacker now takes the hash-code embedded in the recovered message representative and, using the fact that the hash-function is weak, discovers a non-recoverable message part, which, when combined with the recoverable part embedded in the message representative, hashes to the desired hash-code. That is, the attacker can forge a new signature with a 'random'  $M_1$ . Hence the inclusion of a hash-function identifier in the message representative does not avoid the need for the verifier to have a secure independent means of knowing which hash-function to use to verify the signature.

This discussion also relates to the choice of the recoverable length  $c^*$  for signature schemes 2 and 3. As described in clause 7.1.2,  $c^*$  shall be chosen so that  $c^* \leq c$ , the capacity of the signature scheme. It is usually desirable to choose  $c^*$  close to  $c$  so as to maximise the length of the recoverable part of the message, and hence minimise the length of the non-recoverable part of the message. It is suggested that  $c^*$  can be chosen to be somewhat less than  $c$  (e.g.  $c-16$ ,  $c-24$  or  $c-80$ , according to the desired level of difficulty), in order to make attacks of the type described above even more difficult.

## Annex D

(informative)

### Examples

This annex contains a total of 12 worked examples of signature production and signature verification for the three schemes defined in this part of ISO/IEC 9796-2, together with two examples of key production.

Clause D.1 contains examples with public exponent equal to 3.

- Clause D.1.1 contains an example of key production.
- Clause D.1.2 contains three examples of signature production and verification, all of which involve total message recovery. There is one example for each of the three schemes defined in this part of ISO/IEC 9796.
- Clause D.1.3 contains three examples of signature production and verification, all of which involve partial message recovery. There is one example for each of the three schemes defined in this part of ISO/IEC 9796.

Clause D.2 contains examples with public exponent equal to 2.

- Clause D.2.1 contains an example of key production.
- Clause D.2.2 contains three examples of signature production and verification, all of which involve total message recovery. There is one example for each of the three schemes defined in this part of ISO/IEC 9796.
- Clause D.2.3 contains three examples of signature production and verification, all of which involve partial message recovery. There is one example for each of the three schemes defined in this part of ISO/IEC 9796.

#### D.1 Examples with public exponent 3

This clause contains examples for which the public key has exponent 3.

##### D.1.1 Example of key production process

The example key has a modulus of  $k = 1024$  bits with public exponent  $v = 3$ .

$p =$  FB961451 995C82F9 527CAAAF B3FB4254 6D00A01D 8B2BDE3D 2E7B8F7D 0C9E781E  
           B7FABFC8 E86E9F6D ACE3435A 9D043A99 93F3E473 D93FA888 D3577906 77A94931  
 $q =$  FF0EAFC4 70585166 A8CD8E90 36E75290 2F32B863 068016B6 A89F2EA3 418882EF  
           6F570122 F92D2E9B EFFF7329 1818F251 BF095D6E 208F93CD CEF4767A 568AB241

The public modulus  $n$  is the product of the secret prime factors  $p$  and  $q$ . Its length is 1024 bits.

$n =$  FAA8ED34 EEF1CE38 D29814B6 EEAA154D C060BB37 EB1A51E8 AB0398DD ADDFD334  
           CB9BE20C 087B1DDF 1F78A397 62B5F20A 7A730086 30913CD2 EE60183D E249DD16  
           9CA4EB3A E0420E51 13D73050 4A73A926 BEFBFF32 C89858DE 5E5B3899 FEC52521  
           04933163 625F2963 5AB8FAA7 AA14C4F3 C0DD2470 DEFCEB39 2429110A 0149A771

The private signature exponent  $s$  is equal to the multiplicative inverse of  $v$  modulo  $\text{lcm}(p-1, q-1)$ .

$s =$  0A71B48C DF4A1342 5E1BAB87 9F471638 92AEB277 A9CBC369 B1CAD109 3C93FE22  
           33267EC0 805A7693 F6A506D0 F9723F6B 1A6F755A ECB0B7DE 1F440102 94186936  
           316AAC4B F39B37BF 6105DFA0 AEA60B82 C17306F2 179F2ED4 704D5A6F BCB141C0  
           C9380F5A 500823CE 67E8ED81 7F8A5100 59E9541B 498C91F4 1ABE8C10 6220E72B

### D.1.2 Examples with total recovery

Three examples of signature production and verification are provided, one for each of the three schemes.

### D.1.2.1 Example of signature scheme 1

This example uses dedicated hash-function 3 from ISO/IEC 10118-3 (otherwise known as SHA-1).

#### D.1.2.1.1 The signature process

The message to be signed is the following string of 64 ASCII-coded characters.

In hexadecimal, the message  $M$  is the following octet string of length 64, i.e. 512 bits.

**M =** 61626364 62636465 63646566 64656667 65666768 66676869 6768696A 68696A6B  
 696A6B6C 6A6B6C6D 6B6C6D6E 6C6D6E6F 6D6E6F70 6E6F7071 6F707172 70717273

The 160 bits of the hash-code are computed by applying SHA-1 to the 512 bits of  $M$ .

*H* = 79EA0C76 F0056373 FFD6A5AA D389DD90 8B0C0E94

An identifier in the trailer field indicates the hash function in use; ISO/IEC 10118-3 sets the identifier for dedicated hash-function 3 to the value '33'. Therefore, the trailer field  $T$  consists of the following 16 bits.

$T = 33\text{CC}$

The message is short enough for a total recovery. The 1024 bits of the intermediate string  $S_i$  result from concatenating the two bits of the header equal to '01', the more data bit equal to '0', 332 ( $=1024-512-160-16-4$ ) padding bits equal to '0', the border bit equal to 1, the 512 bits of  $M_1$  ( $=M$ ), the 160 bits of  $H$  and the 16 bits of the trailer field  $T$ . The recoverable string  $S_i$  results from replacing the 82 padding nibbles equal to '0' by 82 nibbles equal to 'B' and also the border nibble equal to '1' by a nibble equal to 'A'.

$S_r =$  4BBBBBBB BBBBBBBB BBBBBBBB BBB~~BBBBBB~~ BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB  
 BBBBBBBB BBBBBBBB BBB~~BA~~6162 63646263 64656364 65666465 66676566 67686667  
 68696768 696A6869 6A6B696A 6B6C6A6B 6C6D6B6C 6D6E6C6D 6E6F6D6E 6F706E6F  
 70716F70 71727071 727379E 0C76F005 6373FFD6 A5AAD389 DD908B0C 0E9433CC

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ .  $I_r$  is raised to the power  $s$  modulo  $n$ . The result is represented by a temporary unsigned positive integer  $t$ .

***t*** = D6369220 6E1FE0A5 7DF603C1 E5EE6025 B4EF2E69 3E8C3C9E BA00057B 40860A35  
FCA66D88 33795AC1 91191515 FE852CAD C80F315C 86142051 ED322775 9F307934  
421D615F 39792C40 1319F233 CFFD18D0 12D17A02 442E5BBF B17DCFC5 654BEF59  
5F500A15 365CD5D0 BD27948E C938F7C3 BA775982 472E8921 7424A74B 868B63A8

Because the above result is greater than  $n/2$ , the signature  $\Sigma = n - t$ .

$\Sigma =$  24725B14 80D1ED93 54A210F5 08BBB528 0B718CCE AC8E1549 F1039362 6D59C8FE  
CEF57483 D501C31D 8E5F8E81 6430C55C B263CF29 AA7D1C81 012DF0C8 431963E2  
5A8789DB A6C8E211 00BD3E1C 7A769056 AC2A8530 8469FD1E ACDD68D4 997935C7  
A543274E 2C025392 9D916618 E0DBCD30 0665CAEE 97CE6217 B00469BE 7ABE43C9

The signed message consists of the 128 octets of the signature alone because  $M_2$  is empty.

#### D.1.2.1.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is raised to the power 3 modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s$  = AEED3179 3336127D 16DC58FB 32EE5992 04A4FF7C 2F5E962C EF47DD21 F2241779  
0FE02650 4CBF6223 63BE4234 FF518FA7 160D9D21 CB2AD86D 87F8B2D7 7AE176AF  
343B83D2 76D7A5E7 A96BC6E5 DF073EBB 528E93C6 5B29EC70 EFEBBCB2 8F54B6B1  
9421C1F2 F0ECB8F1 E84580BD 9D9DD4EE 5D69249A 395217AF 469885FD F2B573A5

Since  $I_s$  is congruent to  $(n - 12) \bmod 16$ , it is replaced by its complement to  $n$ , i.e. the recovered integer is  $I'_s = n - I_s$ .

$I'_r =$  4BBBBBBBBB BBBBBBBBB BBBBBBBBB BBBBBBBBB BBBBBBBBB BBBBBBBBB BBBBBBBBB BBBBBBBBB BBBBBBBBBB  
 BBBBBBBBB BBBBBBBBB BBBBA6162 63646263 64656364 65666465 66676566 67686667  
 68696768 696A6869 6A6B696A 6B6C6A6B 6C6D6B6C 6D6E6C6D 6E6F6D6E 6F706E6F  
 70716F70 71727071 727379EA 0C76F005 6373FFD6 A5AAD389 DD908B0C 0E9433CC

$I'_r$  is represented as an unsigned positive integer by the recovered string  $S'_r$ .

- The leftmost octet of  $S'_r$  is equal to '4B'; it consists of the header equal to '01', the more-data bit equal to '0' (total recovery), one padding bit equal to '0' and one padding nibble equal to 'B'; it is followed by 81 nibbles equal to 'B' and the border nibble equal to 'A'; those 42 octets are removed on the right of  $S'_r$ .
- The rightmost octet of  $S'_r$  is equal to 'CC'; therefore the trailer consists from two octets, and is equal to '33CC'; those two octets are also removed on the right of  $S'_r$ .

The hash function identifier is equal to '33'; therefore the hash function in use is dedicated hash-function 3.

The remaining string of 672 bits is divided into two parts.

- $M_1^*$  consists of the leftmost 512 bits.
- $H'$  consists of the rightmost 160 bits.

**$M_1^*$**  = 61626364 62636465 63646566 64656667 65666768 66676869 6768696A 68696A6B  
696A6B6C 6A6B6C6D 6B6C6D6E 6C6D6E6F 6D6E6F70 6E6F7071 6F707172 70717273

*H' = 79EA0C76 F0056373 FFD6A5AA D389DD90 8B0C0E94*

The recovered message  $M^*$  consists of  $M_1^*$  alone because message recovery is total. Another hash-code  $H''$  is computed by applying SHA-1 to  $M^*$ .

H'' = 79EA0C76 F0056373 FFD6A5AA D389DD90 8B0C0E94

Because the two hash-codes  $H'$  and  $H''$  are identical, the signature  $\Sigma$  is accepted.

### D.1.2.2 Example of signature scheme 2

This example uses dedicated hash-function 1 from ISO/IEC 10118-3 (otherwise known as RIPEMD-160).

### D.1.2.2.1 The ~~IDI~~ signature process

In hexadecimal, the message  $M$  is the following octet string of length 48, i.e., 384 bits.

**M =** FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210

The 160 bits of salt  $S$  are generated

S = 436BCA99 54EC376C 96B79C95 D4B82686 F3494AD3

The 160 bits of the hash-code are computed by applying dedicated hash-function 1 to the binary string of length 768 that results from concatenating the 64 bits of the recoverable message length  $C$ , the 384 (=64+384+160+160) bits of the recoverable message part  $M_1$  ( $=M$ ), the 160 bits of the hash-code of the non-recoverable message part (which is empty)  $h(M_2)$  and the 160 bits of salt  $S$ .  $H = h(C \parallel M_1 \parallel h(M_2) \parallel S)$ .

$H = 50BE9461\ 4DA4AF5F\ 8E78C269\ E0DFA03E\ 027CE74F$

An identifier in the trailer field indicates the hash function in use; ISO/IEC 10118-3 sets the identifier for dedicated hash-function 1 to the value '31'. Therefore, the trailer field  $T$  consists of the following 16 bits.

$T = 31CC$

The message is short enough for a total recovery. The 1024 bits of the intermediate string  $S_i$  result from concatenating the 303 (=1024–384–160–160–16–1) padding bits equal to '0', the border bit equal to 1, the 384 bits of  $M_1$  (=M), the 160 bits of  $L$ , the 160 bits of  $H$ , and the 16 bits of the trailer field  $T$ .

$S_i = 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000$   
 $00000000\ 0001FEDC\ BA987654\ 3210FEDC\ BA987654\ 3210FEDC\ BA987654\ 3210FEDC$   
 $BA987654\ 3210FEDC\ BA987654\ 3210FEDC\ BA987654\ 3210436B\ CA9954EC\ 376C96B7$   
 $9C95D4B8\ 2686F349\ 4AD350BE\ 94614DA4\ AF5F8E78\ C269E0DF\ A03E027C\ E74F31CC$

The recoverable string  $S_r$  results from applying the mask generation function  $MGF1$  to the leftmost 848 (=1024–160–16) bits of  $S_i$ , and the leftmost 1 bit of  $S_r$  is set to '0' as  $\delta = 1$  ( $\delta = (1–1024) \bmod 8$ ).

$S_r = 7BB5D930\ 4572EE04\ BECAE622\ 6939DC6D\ A6F19867\ 8B339668\ B09581DA\ 7DC69063$   
 $CFE49956\ 108754DD\ BC3AF3FF\ A6F562C3\ 6C91DAB4\ BFF8CE66\ 29AC5B1B\ 6C1B524A$   
 $49B7669B\ 549E678C\ ABDAD642\ A565394D\ 7373C4C9\ 4ECADF09\ 08A5C00D\ 0511B9F6$   
 $D78039FC\ 7F4BD793\ 420A50BE\ 94614DA4\ AF5F8E78\ C269E0DF\ A03E027C\ E74F31CC$

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ .  $I_r$  is raised to the power  $s$  modulo  $n$ . The result is represented by the temporary unsigned positive integer  $t$ .

$t = A4958BAD\ DA6AB0F5\ E7F544BB\ 1313DB93\ BB733605\ 3678459A\ 31386D3A\ 9F0A477F$   
 $37B853DF\ 6BBBA87B\ ECAC7CD2\ B19FFACD\ 98B40E82\ 0B638D5F\ 7DDAAE56\ FF198EF6$   
 $AB1002C3\ 76C1FFDE\ 03041201\ FF8E6AF9\ 4AFDF056\ 06E10E32\ F3F69091\ 34864AEB$   
 $D983AAA2\ BD725FCC\ A288DECE\ 27810D34\ 807956DC\ 78F3CFC4\ EA45A8DF\ ADA4226C$

The binary string representing the integer  $t$  as an unsigned positive integer is the signature produced by the alternative signature generation function (see clause A.6)  $\Sigma' = t$ .

Because the above result is greater than  $n/2$ , it is replaced by its complement to  $n$ . The binary string representing that integer as an unsigned positive integer is the signature  $\Sigma = n - t$ .

$\Sigma = 56136187\ 14871D42\ EAA2CFB\ DB9639BA\ 04ED8532\ B4A20C4E\ 79CB2BA3\ 0ED58BB5$   
 $93E38E2C\ 9CBF7563\ 32CC26C4\ B115F73C\ E1BEF204\ 252DAF73\ 708569E6\ E3304E1F$   
 $F194E877\ 69800E73\ 10D31E4E\ 4AE53E2D\ 73FE0EDC\ C1B74AAB\ 6A64A808\ CA3EDA35$   
 $2B0F86C0\ A4ECC996\ B8301BD9\ 8293B7BF\ 4063CD94\ 66091B74\ 39E3682A\ 53A58505$

The signed message consists of the 128 octets of the signature alone because  $M_n$  is empty.

#### D.1.2.2.1 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is raised to the power 3 modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s = 7EF31404\ A97EE034\ 13CD2E94\ 857038E0\ 196F22D0\ 5FE6BB7F\ FA6E1703\ 301942D0$   
 $FBB748B5\ F7F3C901\ 633DAF97\ BBC08F47\ 0DE125D1\ 70986E6C\ C4B3BD22\ 762E8ACC$   
 $52ED849F\ 8BA3A6C4\ 67FC5A0D\ A50E6FD9\ 4B883A69\ 79CD79D5\ 55B5788C\ F9B36B2A$   
 $D12F766\ E31351D0\ 18AEA9E9\ 15B3774F\ 117D95F8\ 1C930A59\ 83EB0E8D\ 19FA75A5$

Since  $I_s$  is congruent to  $(n - 12) \bmod 16$ , it is replaced by its complement to  $n$ , i.e. the recovered integer is  $I'_s = n - I_s$ .

$I'_s = 7BB5D930\ 4572EE04\ BECAE622\ 6939DC6D\ A6F19867\ 8B339668\ B09581DA\ 7DC69063$   
 $CFE49956\ 108754DD\ BC3AF3FF\ A6F562C3\ 6C91DAB4\ BFF8CE66\ 29AC5B1B\ 6C1B524A$   
 $49B7669B\ 549E678C\ ABDAD642\ A565394D\ 7373C4C9\ 4ECADF09\ 08A5C00D\ 0511B9F6$   
 $D78039FC\ 7F4BD793\ 420A50BE\ 94614DA4\ AF5F8E78\ C269E0DF\ A03E027C\ E74F31CC$

$I'_i$  is represented as an unsigned positive integer by the recovered string  $S'_i$ . The mask generation function  $MGF1$  applied to the leftmost 848 (=1024–160–16) bits of  $S'_i$ , thus providing the recovered intermediate string  $S'_i$ .

$S'_i = 80000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000$   
 $00000000 0001FEDC BA987654 3210FEDC BA987654 3210FEDC BA987654 3210FEDC$   
 $BA987654 3210FEDC BA987654 3210FEDC BA987654 3210436B CA9954EC 376C96B7$   
 $9C95D4B8 2686F349 4AD350BE 94614DA4 AF5F8E78 C269E0DF A03E027C E74F31CC$

$S'_i$  represents the recovered intermediate string processed as follows.

- The leftmost one bit of  $S'_i$  is set to '0' as  $\delta = 1$  ( $\delta = (1-1024) \bmod 8$ ). The leftmost 37 octets of the resulting binary string are equal to '0'; it is followed by the border octet '01'; those 38 octets are removed on the left of  $S'_i$ .
- The rightmost octet of  $S'_i$  is equal to 'CC'; therefore the trailer consists from two octets, and is equal to '31CC'; those two octets are also removed on the right of  $S'_i$ .

The hash function identifier is equal to '31'; therefore the hash function in use is dedicated hash-function 1.

The remaining string of 704 bits is divided into three parts.

- $M_1^*$  consists of the leftmost 384 bits.
- $S^*$  consists of the rightmost 160 bits.
- $H'$  consists of the rightmost 160 bits.

$M_1^* = FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210$   
 $FEDCBA98 76543210 FEDCBA98 76543210$

$S^* = 436BCA99 54EC376C 96B79C95 D4B82686 F3494AD3$

$H' = 50BE9461 4DA4AF5F 8E78C269 E0DFA03E 027CE74F$

The recovered message  $M^*$  consists of  $M_1^*$  alone because message recovery is total. Another hash-code  $H''$  is computed by applying dedicated hash-function 1 to the binary string of length 768 (=64+384+160+160), that results from concatenating the 64 bits of the recovered message length  $C'$ , the 384 bits of the recovered message  $M^*$ , the 160 bits of the hash-code of the non-recoverable message part (which is empty)  $h(M_2)$  and the 160 bits of the recovered salt  $S^*$ .  $H'' = h(C' \parallel M_1 \parallel h(M_2) \parallel S^*)$ .

$H'' = 50BE9461 4DA4AF5F 8E78C269 E0DFA03E 027CE74F$

Because the two hash-codes  $H'$  and  $H''$  are identical, the signature  $\Sigma$  is accepted.

#### D.1.2.3 Example of signature scheme 3

This example uses dedicated hash-function 3 from ISO/IEC 10118-3 (otherwise known as SHA-1).

##### D.1.2.3.1 The signature process

The message to be signed is empty, i.e. a binary string of length zero.

Because this signature scheme is of deterministic type, a zero length salt value  $S$  is selected.

The 160 bits of the hash-code  $H$  are computed by applying dedicated hash-function 3 to the binary string of length 224 (=64+160), that results from concatenating the 64 bits of the recoverable message length  $C$  and the 160 bits of the hash-code of the non-recoverable message part (which is empty)  $h(M_2)$ .  $H = h(C \parallel h(M_2))$ .

$H = A35D1688 A60AC69F D53E4442 8BFD380E 94DB9176$

The hash function in use is implicitly known. Therefore, the trailer field  $T$  consists of a single octet.

$T = BC$

The message is short enough for a total recovery. The 1024 bits of the intermediate string  $S_i$  result from concatenating the 855 (=1024–160–8–1) padding bits equal to '0', the border bit equal to 1, the 160 bits of  $H$ , and the 8 bits of the trailer field  $T$ .

$S_i =$  00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000  
 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000  
 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000  
 00000000 00000000 000001A3 5D1688A6 0AC69FD5 3E44428B FD380E94 DB9176BC

The recoverable string  $S_r$  results from applying the mask generation function  $MGF1$  to the leftmost 856 (=1024–160–8) bits of  $S_i$ , and the leftmost 1 bit of  $S_r$  is set to '0' as  $\delta = 1$  ( $\delta = (1–1024) \bmod 8$ ).

$S_r =$  7CCB5422 2079C84C 343B0AB1 6307273B 36359229 BD3DFDEC A9FE8054 AD1EF319  
 44758A67 3B7C70C2 FACB6FE9 12690EE2 6DF58975 585A78C2 723F0C71 50535C80  
 8F0868F6 CA94F36C FB079FBB 9126286D 5EECA3CA ACA12593 033A0D64 136A7A72  
 D605080A 6CF68B6D DA0AE6A3 5D1688A6 0AC69FD5 3E44428B FD380E94 DB9176BC

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ .  $I_r$  is raised to the power  $s$  modulo  $n$ . The result is represented by the temporary unsigned positive integer  $t$ .

$t =$  F9DD9F72 FAB4AFFC ED3B0538 C5848B27 756AC50C B2890F4C BC268D96 C5E91EE8  
 8E3B058F 2EF6585F EF5323CA 4E2C308C C6140CF5 F5357960 5B3BF0CC 621082EB  
 77F4A42D 3567355E AA151FB4 652BAFFE 58A4B310 7A064669 FD4177C8 D79F5DE5  
 EEC562FF A2D0F5D9 C409AEA0 D5B9F8DF 493AF2F1 8F91D828 CE32C4CC 35C13113

The binary string representing the integer  $t$  as an unsigned positive integer is the signature produced by the alternative signature generation function (see clause A.6)  $\Sigma' = t$ .

Because the above result is greater than  $n/2$ , the signature  $\Sigma = n - t$ .

$\Sigma =$  00CB4DC1 F43D1E3B E55D0F7E 29258A26 4AF5F62B 3891429B EEDD0B46 E7F6B44C  
 3D60DC7C D984C57F 30257FCB 1489C17D B45EF390 3B5BC372 93242771 80395A2B  
 24B0470D AADAD8F2 69C2109B E547F928 60574C22 4E921274 6119C0D1 2725C73B  
 15CDCE63 BF8E3389 96AF4C06 D45ACC14 77A2317F 4F6B1310 55F64C3D CB88765E

The signed message consists of the 128 octets of the signature alone because  $M_2$  is empty.

#### D.1.2.3.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is raised to the power 3 modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s =$  7DDD9912 CE7805EC 9E5D0A05 8BA2EE12 8A2B290E 2DDC53FC 01051889 00C0E01B  
 872657A4 CCFEAD1C 24AD33AE 504CE328 0C7D7710 D836C410 7C210BCC 91F68096  
 0D9C8244 15AD1AE4 18CF9094 B94D80B9 600F5B68 1BF7334B 5B212B35 EB5AAAAE  
 2E8E2958 F5689DF5 80AE1404 4CFE3C4D B616849B A0B8A8AD 26F10275 25B830B5

Since  $I_s$  is congruent to  $(n - 12) \bmod 16$ , the recovered integer is  $I_r' = n - I_s$ .

$I_r' =$  7CCB5422 2079C84C 343B0AB1 6307273B 36359229 BD3DFDEC A9FE8054 AD1EF319  
 44758A67 3B7C70C2 FACB6FE9 12690EE2 6DF58975 585A78C2 723F0C71 50535C80  
 8F0868F6 CA94F36C FB079FBB 9126286D 5EECA3CA ACA12593 033A0D64 136A7A72  
 D605080A 6CF68B6D DA0AE6A3 5D1688A6 0AC69FD5 3E44428B FD380E94 DB9176BC

$I_r'$  is represented as an unsigned positive integer by the recovered string  $S_r'$ . The mask generation function  $MGF1$  applied to the leftmost 856 (=1024–160–8) bits of  $S_r'$ , thus providing the recovered intermediate string  $S_i'$ .

$S_i' =$  00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000  
 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000  
 00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000  
 00000000 00000000 000001A3 5D1688A6 0AC69FD5 3E44428B FD380E94 DB9176BC

$S_i'$  represents the recovered intermediate string processed as follows.

- The leftmost one bit of  $S'_i$  is set to '0' as  $\delta = 1$  ( $\delta = (1-1024) \bmod 8$ ). The leftmost 106 octets of the resulting binary string are equal to '0'; it is followed by the border octet '01'; those 107 octets are removed on the left of  $S'_i$ .
- The rightmost octet of  $S'_i$  is equal to 'BC'; this octet is also removed on the right of  $S'_i$ .

Because the trailer is equal to 'BC', the hash function in use is implicitly known: dedicated hash-function 3 in this example.

The remaining string of 160 bits is assumed to be the hash-code  $H$  as there is no more data left.

*H'* = A35D1688 A60AC69F D53E4442 8BFD380E 94DB9176

The recovered message  $M'$  is assumed to be empty and, hence, the recovery is total. Another hash-code  $H'$  is computed by applying SHA-1 to the binary string of length 224 (=64+160), that results from concatenating the 64 bits of the recovered message length  $C'$  and the 160 bits of the hash-code of the non-recoverable message part (which is empty)  $h(M_n)$ .  $H = h(C' \parallel h(M_n))$ .

$H'' =$  A35D1688 A60AC69F D53E4442 8BFD380E 94DB9176

Because the two hash-codes  $H$  and  $H'$  are identical, the signature  $\Sigma$  is accepted.

### D.1.3 Examples with partial recovery

Three examples of signature production and verification are provided, one for each of the three schemes.

#### D.1.3.1 Example of signature scheme 1

This example uses dedicated hash-function 1 from ISO/IEC 10118-3 (otherwise known as RIPEMD-160).

### D.1.3.1.1 The signature process

This example illustrates the signature of a message of 132 octets, i.e., 1056 bits.

**M =** FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
FEDCBA98

The 160 bits of the hash-code are computed by applying dedicated hash-function 1 to the 1056 bits of  $M$ .

H = F0EA911A F528FA38 777D4B9A 58B6FDA4 2D7E1999

The hash function in use is implicitly known. Therefore, the trailer field  $T$  consists of the following 8 bits.

$$T = BC$$

The message is too long for being entirely recoverable by the verification process. Therefore, it is divided in two parts.

- $M_1$  consists of the leftmost 848 bits.
- $M_2$  consists of the remaining 208 bits, i.e., 26 octets.

**$M_1 =$**  FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98 76543210 FEDC

**$M_2 =$**  BA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98

The 1024 bits of the intermediate string  $S_r$  result from concatenating the two bits of the header equal to '01', the more data bit equal to '1', four (=1024–848–160–8–4) padding bits equal to '0', the border bit equal to 1, the 848 bits of  $M_1$ , the 160 bits of  $H$  and the 8 bits of the trailer field  $T$ . The recoverable string  $S_r$  results from replacing the border nibble equal to '1' by a nibble equal to 'A'.

$S_r =$  6AFEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCF0 EA911AF5 28FA3877 7D4B9A58 B6FDA42D 7E1999BC

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ .  $I_r$  is raised to the power  $s$  modulo  $n$ . The result is represented by the temporary unsigned positive integer  $t$ .

$t =$  C9DE5B79 67CFD8BE 506749A2 F2E5035C 9C2C5E94 3DD46838 AEF7144E A01283F0  
 95C35FE5 53A87553 AEADBBCE 2B9876EC 14EA5C31 EA11BCC1 F33E5161 7B4B73C2  
 38EB6D4C AA3DF32D 1434E846 E2E74146 E24C7171 D2A0FBED 77E37371 1444360B  
 962A9C27 D9CC2E15 4FE30BEC A3E20B4C OCCF472F 70E64A9C 9FFAA56A 98BC1079

Because the above result is greater than  $n/2$ , the signature  $\Sigma = n - t$ .

$\Sigma =$  30CA91BB 8721F57A 8230CB13 FBC511F1 24345CA3 AD45E9AF FC0C848F 0DCD4F44  
 35D88226 B4D2A88B 70CAE7C9 371D7B1E 6588A454 467F8010 FB21C6DC 66FE6954  
 63B97DEE 36041B23 FFA24809 678C67DF DCAF8DC0 F5F75CF0 E677C528 EA80EF15  
 6E68953B 8892FB4E 0AD5EEBB 0632B9A7 B40DDD41 6E16A09C 842E6B9F 688D96F8

The signed message consists of the 128 octets of the signature  $\Sigma$  together with the 26 octets of the non-recoverable message  $M_2$ , i.e., only 22 octets more than the message  $M$ .

#### D.1.3.1.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is raised to the power 3 modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s =$  8FAA107A 567B7A06 C19937FC 5633C11B AF61DE7D 52A3FDB6 9A04BC23 15697F02  
 BA9D0551 7004C9AD 0E79C6DC CA3F9DD8 697423CB 981AE8A0 DD613B83 49D388E4  
 8BA60E80 47CBBA1F 02D85395 B1FD54F4 ADFD2278 302204AC 4D5C5BDF 664ED0EE  
 F39454A8 C9E8D531 49BA1DB6 BF83A9FE 97E2EBF9 61B150E0 6D2B6CDC 83300DB5

Since  $I_s$  is congruent to  $(n - 12) \bmod 16$ , it is replaced by its complement to  $n$ , i.e. the recovered integer is  $I'_s = n - I_s$ .

$I'_s =$  6AFEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCF0 EA911AF5 28FA3877 7D4B9A58 B6FDA42D 7E1999BC

$I'_s$  is represented as unsigned positive integer by the recovered string  $S'_s$ .

- The leftmost octet of  $S'_s$  is equal to '6A'; it consists of the header equal to '01', the more-data bit equal to '1' (partial recovery), one padding bit equal to '0' and the border nibble equal to 'A'; this octet is removed on the left of  $S'_s$ .
- The rightmost octet of  $S'_s$  is equal to 'BC'; this octet is also removed on the right of  $S'_s$ .

Because the trailer is equal to 'BC', the hash function in use is implicitly known: dedicated hash-function 1 in this example.

The remaining string of 1008 bits is divided into two parts.

- $M_1^*$  consists of the leftmost 848 bits.
- $H'$  consists of the rightmost 160 bits.

$$M_1^* = \begin{array}{cccccccccc} \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 \\ \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 \\ \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 \\ \text{FEDCBA98} & 76543210 & \text{FEDC} \end{array}$$

*H' = F0EA911A F528FA38 777D4B9A 58B6FDA4 2D7E1999*

Because the recovery is partial, the recovered message  $M^*$  consists of the concatenation of  $M_1^*$  and  $M_2^*$ , the recovered and non-recoverable parts respectively.

Another hash-code  $H''$  is computed by applying dedicated hash-function 1 to  $M^*$ .

H'' = F0EA911A F528FA38 777D4B9A 58B6FDA4 2D7E1999

Because the two hash-codes  $H$  and  $H'$  are identical, the signature  $\Sigma$  is accepted

### D.1.3.2 Example of signature scheme 2

This example uses dedicated hash-function 3 from ISO/IEC 10118-3 (otherwise known as SHA-1).

### D.1.3.2.1 The signature process

The message to be signed is the following string of 112 ASCII-coded characters:

In hexadecimal, the message  $M$  is the following octet string of length 112 octets, i.e., 896 bits:

**M =** 61626364 62636465 63646566 64656667 65666768 66676869 6768696A 68696A6B  
696A6B6C 6A6B6C6D 6B6C6D6E 6C6D6E6F 6D6E6F70 6E6F7071 6F707172 70717273  
71727374 72737475 73747576 74757677 75767778 76777879 7778797A 78797A61  
797A6162 7A616263 61626364 62636465

The 160 bits of salt  $S$  are generated.

S = 4C95C1B8 7A1DE8AC C193C14C F3147FE9 C6636078

The message is too long to be entirely recoverable by the verification process. Therefore, it is divided in two parts.

- $M_1$  consists of the leftmost 688 bits.
- $M_2$  consists of the remaining 208 bits, i.e., 26 octets.

**M<sub>1</sub>** = 61626364 62636465 63646566 64656667 65666768 66676869 6768696A 68696A6B  
696A6B6C 6A6B6C6D 6B6C6D6E 6C6D6E6F 6D6E6F70 6E6F7071 6F707172 70717273  
71727374 72737475 73747576 74757677 75767778 7677

**M<sub>2</sub>** = 7879 7778797A 78797A61 797A6162 7A616263 61626364 62636465

The 160 bits of the hash-code are computed by applying dedicated hash-function 3 to the binary string of length 1072 (=64+688+160+160), that results from concatenating the 64 bits of the recoverable message length  $C$ , the 688 bits of the recoverable message part  $M_r$ , 160 bits of the hash-code of the non-recoverable message part  $h(M_n)$ , and 160 bits of salt  $S$ .  $H = h(C \parallel M_1 \parallel h(M_2) \parallel S)$ .

*H* = 16671F61 4F2954A8 6E51CB81 102A3D47 E2C11EBD

The hash function in use is implicitly known. Therefore, the trailer field  $T$  consists of the following 8 bits.

$T = \text{BC}$

The 1024 bits of the intermediate string  $S_i$  result from concatenating the 7 (=1024–688–160–160–8–1) padding bits equal to '0', the border bit equal to 1, the 688 bits of  $M_1$ , the 160 bits of  $L$ , the 160 bits of  $H$ , and the 8 bits of the trailer field  $T$ .

$S_i = 01616263\ 64626364\ 65636465\ 66646566\ 67656667\ 68666768\ 69676869\ 6A68696A$   
 $6B696A6B\ 6C6A6B6C\ 6D6B6C6D\ 6E6C6D6E\ 6F6D6E6F\ 706E6F70\ 716F7071\ 72707172$   
 $73717273\ 74727374\ 75737475\ 76747576\ 77757677\ 7876774C\ 95C1B87A\ 1DE8ACC1$   
 $93C14CF3\ 147FE9C6\ 63607816\ 671F614F\ 2954A86E\ 51CB8110\ 2A3D47E2\ C11EBDBC$

The recoverable string  $S_r$  results from applying the mask generation function  $MGF1$  to the leftmost 856 (=1024–160–8) bits of  $S_i$ , and the leftmost 1 bit of  $S_r$  is set to '0' as  $\delta = 1$  ( $\delta = (1–1024) \bmod 8$ ).

$S_r = 390871A1\ 2B83F417\ 63782F59\ BB700DD4\ 63C071B6\ 98C7D152\ 9B8616A9\ B72DF9DE$   
 $B899BFA8\ C3839DD0\ 903CEF72\ A9C18496\ 64129992\ 6FE8D3F8\ FDA1D07D\ 2251EAAD$   
 $34017F7D\ C66DD60F\ D3F0014D\ 23CA2D2E\ 43383F30\ 9724B846\ 2529C5FE\ 73205FB5$   
 $D1FCC8CF\ D18C687F\ B9E35616\ 671F614F\ 2954A86E\ 51CB8110\ 2A3D47E2\ C11EBDBC$

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ .  $I_r$  is raised to the power  $s$  modulo  $n$ . The result is represented by the temporary unsigned positive integer  $t$ .

$t = 92ACA17F\ 28426177\ 1E4A1313\ C0510483\ 8C3CC91C\ 1CB6F576\ CF95090A\ 5FDEA51E$   
 $3C189F65\ E6BA3F28\ 4268B4FF\ 2363B3B9\ 12D023A9\ 1C96541A\ C1F9E60E\ 58F6B3DA$   
 $8DEB1B69\ 41792AA6\ 341DB184\ 88366A5E\ 1E18DBBA\ E4A2E390\ 77A2B4FE\ 1DFB34A2$   
 $CCAD1812\ C4AFFAF5\ 5570855A\ AEB685DA\ 2E1F124F\ F70F529F\ ED02F515\ BFD572AE$

The binary string representing the integer  $t$  as an unsigned positive integer is the signature produced by the alternative signature generation function (see clause A.6)  $\Sigma = t$ .

Because the above result is greater than  $n/2$ , the signature  $\Sigma = n - t$ .

$\Sigma = 67FC4BB5\ C6AF6CC1\ B44E01A3\ 2E5910CA\ 3423F21B\ CE635C71\ DB6E8FD3\ 4E012E16$   
 $8F8342A6\ 21C0DEB6\ DD0FEE98\ 3F523E51\ 67A2DCDD\ 13FAE8B8\ 2C66322F\ 8953293C$   
 $0EB9CFD1\ 9EC8E3AA\ DFB97ECB\ C23D3EC8\ A0E32377\ E3F5754D\ E6B8839B\ E0C9F07E$   
 $37E61950\ 9DAF2E6E\ 0548754C\ FB5E3F19\ 92BE1220\ E7ED9899\ 37261BF4\ 417434C3$

The signed message consists of the 128 octets of the signature  $\Sigma$  together with the 26 octets of the non-recoverable message  $M_2$ , i.e., only 42 octets more than the message  $M$ .

#### D.1.3.2.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is raised to the power 3 modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s = C1A07B93\ C36DDA21\ 6F1FE55D\ 333A0779\ 5CA04981\ 52528096\ 0F7D8233\ F6B1D956$   
 $13022263\ 44F7800E\ 8F3BB424\ B8F46D74\ 166066F3\ C0A868D9\ F0BE47C0\ BFF7F269$   
 $68A36BBB\ 19D43841\ 3FE72F03\ 26A97BF8\ 7BC3C002\ 3173A098\ 3931729B\ 8BA4C56B$   
 $32966893\ 90D2C0E3\ A0D5A491\ 42F563A4\ 97887C02\ 8D316A28\ F9EBC927\ 402AE9B5$

Since  $I_s$  is congruent to  $(n - 12) \bmod 16$ , the recovered integer is  $I'_r = n - I_s$ .

$I'_r = 390871A1\ 2B83F417\ 63782F59\ BB700DD4\ 63C071B6\ 98C7D152\ 9B8616A9\ B72DF9DE$   
 $B899BFA8\ C3839DD0\ 903CEF72\ A9C18496\ 64129992\ 6FE8D3F8\ FDA1D07D\ 2251EAAD$   
 $34017F7D\ C66DD60F\ D3F0014D\ 23CA2D2E\ 43383F30\ 9724B846\ 2529C5FE\ 73205FB5$   
 $D1FCC8CF\ D18C687F\ B9E35616\ 671F614F\ 2954A86E\ 51CB8110\ 2A3D47E2\ C11EBDBC$

$I'_r$  is represented as an unsigned positive integer by the recovered string  $S'_r$ . The mask generation function  $MGF1$  applied to the leftmost 856 (=1024–160–8) bits of  $S'_r$ , thus providing the recovered intermediate string  $S'_i$ .

$S'_i = 01616263\ 64626364\ 65636465\ 66646566\ 67656667\ 68666768\ 69676869\ 6A68696A$

6B696A6B 6C6A6B6C 6D6B6C6D 6E6C6D6E 6F6D6E6F 706E6F70 716F7071 72707172  
 73717273 74727374 75737475 76747576 77757677 7876774C 95C1B87A 1DE8ACC1  
 93C14CF3 147FE9C6 63607816 671F614F 2954A86E 51CB8110 2A3D47E2 C11EBDBC

$S'$  represents the recovered intermediate string processed as follows.

- The leftmost one bit of  $S'$  is set to '0' as  $\delta = 1$  ( $\delta = (1-1024) \bmod 8$ ). The leftmost 7 bits of the resulting binary string are equal to '0'; it is followed by the border bit '1'; that 1 octet is removed on the left of  $S'$ .
- The rightmost octet of  $S'$  is equal to 'BC'; this octet is also removed on the right of  $S'$ .

Because the trailer field  $T$  is equal to 'BC', the hash function in use is implicitly known: dedicated hash-function 3 in this example.

The remaining string of 1008 bits is divided into three parts.

- $M_1^*$  consists of the leftmost 688 bits.
- $S^*$  consists of the rightmost 160 bits.
- $H'$  consists of the rightmost 160 bits.

$M_1^* =$  61626364 62636465 63646566 64656667 65666768 66676869 6768696A 68696A6B  
 696A6B6C 6A6B6C6D 6B6C6D6E 6C6D6E6F 6D6E6F70 6E6F7071 6F707172 70717273  
 71727374 72737475 73747576 74757677 75767778 7677

$S^* =$  4C95C1B8 7A1DE8AC C193C14C F3147FE9 C6636078

$H' =$  16671F61 4F2954A8 6E51CB81 102A3D47 E2C11EBD

Because the recovery is partial, the recovered message  $M^*$  consists of the concatenation of  $M_1^*$  and  $M_2^*$ , the recovered and non-recoverable parts respectively.

$M^* =$  61626364 62636465 63646566 64656667 65666768 66676869 6768696A 68696A6B  
 696A6B6C 6A6B6C6D 6B6C6D6E 6C6D6E6F 6D6E6F70 6E6F7071 6F707172 70717273  
 71727374 72737475 73747576 74757677 75767778 76777879 7778797A 78797A61  
 797A6162 7A616263 61626364 62636465

Another hash-code  $H''$  is computed by applying SHA-1 to the binary string of length 1072 (=64+688+160+160), that results from concatenating the 64 bits of the recovered message length  $C'$ , the 688 bits of the recovered message part  $M_1^*$ , the 160 bits of the hash-code of the non-recoverable message part  $h(M_2^*)$ , and the 160 bits of the recovered salt  $S^*$ .  $H'' = h(C' \parallel M_1^* \parallel h(M_2^*) \parallel S^*)$ .

$H'' =$  16671F61 4F2954A8 6E51CB81 102A3D47 E2C11EBD

Because the two hash-codes  $H'$  and  $H''$  are identical, the signature  $\Sigma$  is accepted.

### D.1.3.3 Example of signature scheme 3

This example uses dedicated hash-function 3 from ISO/IEC 10118-3 (otherwise known as SHA-1).

#### D.1.3.3.1 The signature process

This example illustrates the signature of a message of 132 octets, i.e., 1056 bits.

$M =$  FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98

Because this signature scheme is of deterministic type, a zero length salt value  $S$  is selected.

The message is too long for being entirely recoverable by the verification process. Therefore, it is divided in two parts.

- $M_1$  consists of the leftmost 840 bits.
- $M_2$  consists of the remaining 216 bits, i.e., 27 octets.

$M_1 =$  FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
 FEDCBA98 76543210 FE

$M_2 =$  DCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98

The 160 bits of the hash-code  $H$  are computed by applying dedicated hash-function 3 to the binary string of length 1064 (=64+840+160), that results from concatenating the 64 bits of the recoverable message length  $C$ , the 840 bits of the recoverable message part  $M_1$ , and the 160 bits of the hash-code of the non-recoverable message part  $h(M_2)$ .  
 $H = h(C \parallel M_1 \parallel h(M_2))$ .

$H =$  E30A9CB8 F10DC3C8 1897D9E8 D394555A AC6DEC79

An identifier in the trailer field  $T$  indicates the hash function in use, i.e. dedicated hash-function 3; ISO/IEC 10118-3 sets the identifier for this hash-function to the value '33'. Therefore, the trailer field  $T$  consists of the following 16 bits.

$T =$  33CC

The 1024 bits of the intermediate string  $S_i$  result from concatenating the 7 (=1024–840–160–16–1) padding bits equal to '0', the border bit equal to 1, 840 bits of the recoverable message part  $M_1$ , 160 bits of the hash-code of the non-recoverable message part  $h(M_2)$ , and the 16 bits of the trailer field  $T$ .

$S_i =$  01FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432 10FEDCBA 98765432  
 10FEDCBA 98765432 10FEE30A 9CB8F10D C3C81897 D9E8D394 555AAC6D EC7933CC

The recoverable string  $S_r$  results from applying the mask generation function  $MGF1$  to the leftmost 848 (=1024–160–16) bits of  $S_i$ , and the leftmost 1 bit of  $S_r$  is set to '0' as  $\delta = 1$  ( $\delta = (1–1024) \bmod 8$ ).

$S_r =$  1E1F9F67 4356F609 0062DEA3 FC994589 8259AE44 7F4ACAD2 0655D646 0435C851  
 ED9D1754 837A1269 1886C244 DED123EB 470510BC 459AD416 99310030 C824907D  
 0DC63B7F 422008FA 4D68E912 0DFDD534 8FAD5056 DD7A43F1 BB38E64A EEDF14CD  
 2549A7F6 B1869D77 C4E5E30A 9CB8F10D C3C81897 D9E8D394 555AAC6D EC7933CC

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ .  $I_r$  is raised to the power  $s$  modulo  $n$ . Being less than  $n/2$  the result is kept. The binary string representing that integer as an unsigned positive integer is the signature  $\Sigma$ .

$\Sigma =$  30147ECB 074705DD F33EF765 D0EE1017 D5535AB3 9A7727C4 D8D4DC42 42C693BD  
 1FB544EC AE2323D1 185BED05 C8AA5F69 9D3AAED4 1FC3ECF9 DF297A61 56D6BC86  
 5196A619 806E3FDF F8A8416D 2984EF9E 33940013 4A6D1712 2FCF0946 783AEBD4  
 6F11397E 66863E74 28F4542D E2AE8A30 7355633F 380F937B 308C149F 14194487

In this example the signature produced by the alternative signature generation function (see clause A.6) is also the binary string  $\Sigma$ , i.e.  $\Sigma' = \Sigma$ .

The signed message consists of the 128 octets of the signature  $\Sigma$  together with the 27 octets of the non-recoverable message  $M_n$ , i.e., only 23 octets more than the message  $M$ .

#### D.1.3.3.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is raised to the power 3 modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s = 1E1F9F67\ 4356F609\ 0062DEA3\ FC994589\ 8259AE44\ 7F4ACAD2\ 0655D646\ 0435C851$   
 $ED9D1754\ 837A1269\ 1886C244\ DED123EB\ 470510BC\ 459AD416\ 99310030\ C824907D$   
 $0DC63B7F\ 422008FA\ 4D68E912\ 0DFDD534\ 8FAD5056\ DD7A43F1\ BB38E64A\ EEDF14CD$   
 $2549A7F6\ B1869D77\ C4E5E30A\ 9CB8F10D\ C3C81897\ D9E8D394\ 555AAC6D\ EC7933CC$

Since  $I_s$  is here congruent to 12 mod 16, the recovered integer is  $I'_r = I_s$ .

$I'_r$  is represented as an unsigned positive integer by the recovered string  $S'_r$ . The mask generation function  $MGF1$  applied to the leftmost 848 (=1024–160–16) bits of  $S'_r$ , thus providing the recovered intermediate string  $S'_i$ .

$S'_i = 81FEDCBA\ 98765432\ 10FEDCBA\ 98765432\ 10FEDCBA\ 98765432\ 10FEDCBA\ 98765432$   
 $10FEDCBA\ 98765432\ 10FEDCBA\ 98765432\ 10FEDCBA\ 98765432\ 10FEDCBA\ 98765432$   
 $10FEDCBA\ 98765432\ 10FEDCBA\ 98765432\ 10FEDCBA\ 98765432\ 10FEDCBA\ 98765432$   
 $10FEDCBA\ 98765432\ 10FEE30A\ 9CB8F10D\ C3C81897\ D9E8D394\ 555AAC6D\ EC7933CC$

$S'_i$  represents the recovered intermediate string processed as follows.

- The leftmost one bit of  $S'_i$  is set to '0' as  $\delta = 1$  ( $\delta = (1-1024) \bmod 8$ ). The leftmost 7 bits of the resulting binary string are equal to '0'; it is followed by the border bit '1'; that 1 octet is removed on the left of  $S'_i$ .
- The rightmost octet of  $S'_i$  is equal to 'CC'; therefore the trailer consists from two octets, and is equal to '33CC'; those two octets are also removed on the right of  $S'_i$ .

The hash function identifier is equal to '33'; therefore the hash function in use is dedicated hash-function 3.

The remaining string of 1000 bits is divided into two parts.

- $M_1^*$  consists of the leftmost 840 bits.
- $H'$  consists of the rightmost 160 bits.

$M_1^* = FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98\ 76543210\ FE$

$H' = E30A9CB8\ F10DC3C8\ 1897D9E8\ D394555A\ AC6DEC79$

Because the recovery is partial, the recovered message  $M^*$  consists of the concatenation of  $M_1^*$  and  $M_2^*$ , the recovered and non-recoverable parts respectively.

$M^* = FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210\ FEDCBA98\ 76543210$   
 $FEDCBA98$

Another hash-code  $H''$  is computed by applying SHA-1 to the binary string of length 1064 (=64+840+160), that results from concatenating the 64 bits of the recovered message length  $C'$ , the 840 bits of the recovered message part  $M_1^*$ , and the 160 bits of the hash-code of the non-recoverable message part  $h(M_2^*)$ .  $H'' = h(C' \parallel M_1^* \parallel h(M_2^*))$ .

$H'' = E30A9CB8\ F10DC3C8\ 1897D9E8\ D394555A\ AC6DEC79$

Because the two hash-codes  $H'$  and  $H''$  are identical, the signature  $\Sigma$  is accepted.

## D.2 Examples with public exponent 2

This clause contains examples for which the public key has exponent 2.

## D.2.1 Example of key production process

The example key has a modulus of  $k = 1024$  bits with public exponent  $v = 2$ . Because the public verification exponent  $v$  is even, one secret prime factor is congruent to 3 modulo 8 and the other one to 7 mod 8.

$p =$  F69AD66B F97E4CCC B4A76FD3 1F43871D C71100CA F9256C3D BE98CC23 BEC06324  
A2282D3C CFCAF00B 0E7492C0 1FB19CE5 0F73EEFD 1A08B0AE 6756E7DF 5670D69B

$q =$  C41DB9CC D8777062 2BEA8836 1E49AFA2 B5B6CBD0 28479585 472150A1 96C65E89  
C2114580 FDE60F6B E12CA9DD A370A3EA 74D33B52 8EB791A9 0FD52818 3D8F612F

The public modulus  $n$  is the product of the secret prime factors  $p$  and  $q$ . Its length is 1024 bits.

$n =$  BCEB2EB0 2E1C8E99 99BC9603 F8F91DA6 084EA6E7 C75BD18D D0CDBEDB 21DA29F1  
9E731125 9DB0D190 B1920186 A8126B58 2D13ABA6 9958763A DA8F79F1 62C7379D  
6109D2C9 4AA2E041 B383A74B BF17FFCC 145760AA 8B58BE3C 00C52BA3 BD05A9D0  
BE5BA503 E6721FC4 066D37A8 9BF072C9 7BABB26C F6B29633 043DB474 6F9D2175

The private signature exponent  $s$  is equal to the multiplicative inverse of  $v \bmod \text{lcm}(p-1, q-1)/2$ .

$s =$  029FB5FB 55F94917 7777F3DC 7FE703F7 A3ABC251 70FDB83E 6A02DB8A 2794CECE  
05C19920 85BEE677 57CCB1CC 8972089A 1D120D0C FB04C8C0 D141FE23 5A42C453  
F0883D5E 73742EB5 98435B52 B393B491 F053C59C A8950D48 CA990ADF 888C6DE4  
085CEB5D 6B02AEAB BCC2D543 B4C9F995 3FE16572 2F4E0846 9AD92248 D8622DEA

## D.2.2 Examples with total recovery

Three examples of signature production and verification are provided, one for each of the three schemes.

### D.2.2.1 Example of signature scheme 1

This example uses dedicated hash-function 3 from ISO/IEC 10118-3 (otherwise known as SHA-1).

#### D.2.2.1.1 The signature process

In hexadecimal, the message  $M$  is the following octet string of length 48, i.e., 384 bits.

$M =$  FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210 FEDCBA98 76543210  
FEDCBA98 76543210 FEDCBA98 76543210

The 160 bits of the hash-code are computed by applying dedicated hash-function 3 to the 384 bits of  $M$ .

$H =$  85DCC7FC 51371637 5A059D02 5439FCD9 25C828AC

The hash function in use is implicitly known. Therefore, the trailer field  $T$  consists of the following 8 bits.

$T =$  BC

The message is short enough for a total recovery. The 1024 bits of the intermediate string  $S_i$  result from concatenating the two bits of the header equal to '01', the more data bit equal to '0', 468 ( $=1024-384-160-8-4$ ) padding bits equal to '0', the border bit equal to 1, the 384 bits of  $M_1$  ( $=M$ ), the 160 bits of  $H$  and the 8 bits of the trailer field  $T$ . The recoverable string  $S_r$  results from replacing the 116 padding nibbles equal to '0' by the 116 nibbles equal to 'B' and also the border nibble equal to '1' by a nibble equal to 'A'.

$S_r =$  4BBBBBBB BBBB  
BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB BBBB  
543210FE DCBA9876 543210FE DCBA9876 543210FE DCBA9876 543210FE DCBA9876  
543210FE DCBA9876 54321085 DCC7FC51 3716375A 059D0254 39FCD925 C828ACBC

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ . Because the Jacobi symbol of  $I_r$  with respect to  $n$  equal to 1, the result is kept.  $I_r$  is raised to the power  $s$  modulo  $n$ . Being less than  $n/2$  the result is kept. The binary string representing that integer as an unsigned positive integer is the signature  $\Sigma$ .

|            |          |          |          |          |          |          |          |          |
|------------|----------|----------|----------|----------|----------|----------|----------|----------|
| $\Sigma =$ | 0C0C62D3 | 523F2DA3 | 972679D0 | 348D9A50 | 38E93AE3 | D19E97DF | 875DCC04 | 6B2637DB |
|            | CE7D4CCC | 5967529A | B96D27B6 | D9B41F54 | 56E65EEA | 328FDB7D | AE6F4E7D | A0CFC1CF |
|            | F8AB5A80 | CC7C9B9F | 487EC2B5 | 90CBC2F3 | 1AFDC5CF | 9C3478B9 | 3C46D575 | A0E08D21 |
|            | D965A9C4 | FCAFE356 | 2D64B1C3 | 0706AF0D | 43288156 | DA3FF990 | CB040D5C | 0863F262 |

The signed message consists of the 128 octets of the signature alone because  $M_2$  is empty.

#### D.2.2.1.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is squared modulo  $n$ , thus providing the resulting integer  $I_s$ .

$I_s =$  712F72F4 7260D2DD DE00DA48 3D3D61EA 4C92EB2C 0BA015D2 1512031F 661E6E35  
E2B75569 E1F515D4 F5D645CA EC56AF9C 7157EFEA DD9CBA7F 1ED3BEF2 860C9F27  
0CD7C1CA 6DE847CB 5F51964C E25D6755 C0254FAB AE9E25C5 AC931AA4 E04B115A  
6A299405 09B7874D B23B2722 BF287678 44957B12 F11593DE CA40DB4E A77474B9

The verification process does not involve the Jacobi symbol. Because the least significant three bits of the resulting integer  $I_s$  are equal to '001',  $I_r = n - I_s$ .

$I'_r =$  4BBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB  
BBBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBBBB BBBBBAFE DCBA9876  
543210FE DCBA9876 543210FE DCBA9876 543210FE DCBA9876 543210FE DCBA9876  
543210FE DCBA9876 54321085 DCC7FC51 3716375A 059D0254 39FCD925 C828ACBC

$I'_r$  is represented as an unsigned positive integer by the recovered string  $S'_r$ .

- The leftmost octet of  $S'_r$  is equal to '4B'; it consists of the header equal to '01', the more-data bit equal to '0' (total recovery), one padding bit equal to '0' and one padding nibble equal to 'B'; it is followed by 115 nibbles equal to 'B' and the border nibble equal to 'A'; those 59 octets are removed on the left of  $S'_r$ .
- The rightmost octet of  $S'_r$  is equal to 'BC'; this octet is also removed on the right of  $S'_r$ .

Because the trailer field  $T$  is equal to 'BC', the hash function in use is implicitly known: dedicated hash-function 3 in this example.

The remaining string of 544 bits is divided into two parts.

- $M_1^*$  consists of the leftmost 384 bits.
- $H'$  consists of the rightmost 160 bits.

$$M_1^* = \begin{matrix} \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 \\ \text{FEDCBA98} & 76543210 & \text{FEDCBA98} & 76543210 \end{matrix}$$

$H' = \text{85DCC7FC } 51371637 \text{ } 5A059D02 \text{ } 5439FCD9 \text{ } 25C828AC$

The recovered message  $M^*$  consists of  $M_1^*$  alone because message recovery is total. Another hash-code  $H''$  is computed by applying SHA-1 to  $M^*$ .

*H'' = 85DCC7FC 51371637 5A059D02 5439FCD9 25C828AC*

Because the two hash-codes  $H$  and  $H''$  are identical, the signature  $\Sigma$  is accepted.

### D.2.2.2 Example of signature scheme 2

This example uses dedicated hash-function 1 from ISO/IEC 10118-3 (otherwise known as RIPEMD-160).

### D.2.2.2.1 The signature process

The message to be signed is empty, i.e. binary string of length zero.

The 160 bits of salt  $S$  are generated.

$S = 61DF870C\ 4890FE85\ D6E3DD87\ C3DCE372\ 3F91DB49$

The 160 bits of the hash-code are computed by applying dedicated hash-function 1 to the binary string of length 384 (=64+160+160), that results from concatenating the 64 bits of the recoverable message length  $C$ , the 160 bits of the hash-code of the non-recoverable message part  $h(M_2)$  and the 160 bits of salt  $S$ .  $H = h(C || h(M_2) || S)$ .

$H = 632E21FD\ 52D2B95C\ 5F7023DA\ 63DE9509\ C01F6C7B$

The hash function in use is implicitly known. Therefore, the trailer field  $T$  consists of the following 8 bits.

$T = BC$

The message is empty, and hence message recovery is total. The 1024 bits of the intermediate string  $S_i$  result from concatenating the 695 (=1024–160–160–8–1) padding bits equal to '0', the border bit equal to 1, the 160 bits of  $S$ , the 160 bits of  $H$ , and the 8 bits of the trailer field  $T$ .

$S_i = 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000$   
 $00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000000$   
 $00000000\ 00000000\ 00000000\ 00000000\ 00000000\ 00000161\ DF870C48\ 90FE85D6$   
 $E3DD87C3\ DCE3723F\ 91DB4963\ 2E21FD52\ D2B95C5F\ 7023DA63\ DE9509C0\ 1F6C7BBC$

The recoverable string  $S_r$  results from applying the mask generation function  $MGF1$  to the leftmost 856 (=1024–160–8) bits of  $S_i$ , and the leftmost 1 bit of  $S_r$  is set to '0' as  $\delta = 1$  ( $\delta = (1–1024) \bmod 8$ ).

$S_r = 73FEAF13\ EB12914A\ 43FE6350\ 22BB4AB8\ 188A8F3A\ BD8D8A9E\ 4AD6C355\ EE920359$   
 $C7F237AE\ 36B1212F\ E947F676\ C68FE362\ 247D27D1\ F298CA93\ 02EB21F4\ A64C26CE$   
 $44471EF8\ C0DFE1A5\ 4606F0BA\ 8E63E87C\ DACA993B\ FA62973B\ 567473B4\ D38FAE73$   
 $AB228600\ 934A9CC1\ D3263E63\ 2E21FD52\ D2B95C5F\ 7023DA63\ DE9509C0\ 1F6C7BBC$

The recoverable integer  $I_r$  is the unsigned positive integer represented by  $S_r$ . Because the Jacobi symbol of  $I_r$  with respect to  $n$  equal to –1, the representative integer is  $J = I_r/2$ .

$J = 39FF5789\ F58948A5\ 21FF31A8\ 115DA55C\ 0C45479D\ 5EC6C54F\ 256B61AA\ F74901AC$   
 $E3F91BD7\ 1B589097\ F4A3FB3B\ 6347F1B1\ 123E93E8\ F94C6549\ 817590FA\ 53261367$   
 $22238F7C\ 606FF0D2\ A303785D\ 4731F43E\ 6D654C9D\ FD314B9D\ AB3A39DA\ 69C7D739$   
 $D5914300\ 49A54E60\ E9931F31\ 9710FEA9\ 695CAE2F\ B811ED31\ EF4A84E0\ 0FB63DDE$

$J$  is raised to the power  $s$  modulo  $n$ . The result is as follows.

$J^s = B6935ACC\ DCABB323\ D7A7125A\ CA86B2E6\ AF7937DE\ 4F523629\ 93B07BF2\ 895A4677$   
 $50553ECE\ 92570E7F\ 975CDB89\ D3EC9487\ CA626E9B\ 4E7FD5A4\ 16ED9C7A\ 9E619DCF$   
 $DC05A5A9\ 4089E593\ 50C9E86B\ 4DD10E5B\ DD709150\ 843D755B\ 057C99F6\ 71330258$   
 $E56474B9\ 6A7A4848\ DC1F4100\ 1603BBAB\ DBA44AE7\ 1A6F8211\ 40137572\ 67C97D0C$

Because the above result is greater than  $n/2$ , the signature  $\Sigma = n - t$ .

$\Sigma = 0657D3E3\ 5170DB75\ C21583A9\ 2E726ABF\ 58D56F09\ 78099B64\ 3D1D42E8\ 987FE37A$   
 $4E1DD257\ 0B59C311\ 1A3525FC\ D425D6D0\ 62B13D0B\ 4AD8A096\ C3A1DD76\ C46599CD$   
 $85042D20\ 0A18FAAE\ 62B9BEE0\ 7146F170\ 36E6CF5A\ 071B48E0\ FB4891AD\ 4BD2A777$   
 $D8F7304A\ 7BF7D77B\ 2A4DF6A8\ 85ECB71D\ A0076785\ DC431421\ C42A3F02\ 07D3A469$

The signed message consists of the 128 octets of the signature alone because  $M_2$  is empty.

### D.2.2.2.2 The verification process

The signature  $\Sigma$  is a binary string representing an unsigned positive integer, which is less than  $n/2$ . That integer is squared modulo  $n$ , thus providing the resulting integer  $I_s$ .