International Standard 3002/2

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Basic quantities in cutting and grinding — Part 2 : Geometry of the active part of cutting tools — General conversion formulae to relate tool and working angles

Définitions de base pour la coupe et la rectification — Partie 2 : Géométrie de la partie active des outils coupants — Formules de conversion générales liant les angles de l'outil en main et les angles en travail

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Foreword

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Basic quantities in cutting and grinding — Part 2 : Geometry of the active part of cutting tools — General conversion formulae to relate tool and working angles

0 Introduction

This part of ISO 3002 presents formulae which can be used to convert tool angles to working angles and vice versa. The formulae are general and can be used for all possible cutting conditions. Tool angles (angles in the tool-in-hand system) and working angles (angles in the tool-in-use system) are defined in ISO 3002/1, together with the sign conventions for these angles.

The tool-in-hand reference system of planes (used to define tool angles) rotates with the cutting tool whenever the orientation of the cutting tool is changed relative to the machine tool. Similarly, the orientation of the tool-in-use reference system of planes (used to define working angles) changes with changes in the resultant cutting direction. It is therefore necessary, in order to relate the tool-in-hand and tool-in-use reference system of planes, to relate both of them to a third reference system of planes, the machine reference system of planes, which does not rotate when the tool is re-oriented or when the resultant cutting direction changes.

The relationship between the tool-in-hand and the machine reference systems of planes defines the setting of the tool in the machine. The relative position of the tool-in-use and the machine reference systems of planes is defined by the motion of the tool relative to the workpiece.

Certain general principles have been taken into consideration for the establishment of the conversion formulae :

a) The definitions of the machine reference system of axes as defined in ISO 841 have been adopted.

b) The origin of each coordinate system is considered to be at the "selected point" on the cutting edge and at a given moment in time.

c) Transformation angles are defined in such a manner as to be computable readily from available workshop data.

1 Scope and field of application

This part of ISO 3002 deals with the establishment and application of the conversion formulae under the following headings :

a) Definitions of the coordinate axes for the tool-in-hand and for the tool-in-use derived from the appropriate planes defined in ISO 3002/1. Definitions for the machine axes and planes are based on ISO 841.

b) Definitions of the setting angles and motion angles.

c) Conversion formulae.

d) Practical examples.

NOTE — In addition to the terms given in the three official ISO languages (English, French, Russian), this part of ISO 3002 gives the equivalent terms in German and Dutch; these terms have been included at the request of Technical Committee ISO/TC 29, and are published under the responsibility of the committee members for Germany, F.R. (DIN) and the Netherlands (NNI). However, only the terms given in the official languages can be considered as ISO terms.

2 Definitions of the coordinates axes

2.1 Tool-in-hand and tool-in-use axes

Axes are defined for the tool-in-hand using the tool-in-hand reference system of planes and for the tool-in-use using the tool-in-use reference system of planes. The axes are defined by the intersection of planes; the appropriate pairs of planes are given in parentheses below.

2.1.1 Tool-in-hand axes

f-Set of coordinate axes : X_f (P_r P_p), Y_f (P_p P_f), Z_f (P_f P_r).

The X_f-axis is positive in a direction which is away from both the reference corner at the tool, (as defined in ISO 3002/1) and the assumed machined surface; the Y_f-axis is positive in a direction opposite to the assumed direction of primary motion; the Z_f-axis is positive in a direction away from the assumed transient surface on the workpiece. [See figures 1 and 2.]

NOTE — The resulting defined axes may conform to either a right hand or a left hand coordinate axis system depending upon the type of tool considered, its intended use and the location of the selected point on the cutting edges.

In a similar way, other sets of tool-in-hand axes can be defined. However, for the practical application of the transformation formulae, only the f-set of coordinate axes is required. Therefore, in what follows, the more general term "tool-in-hand axes" will be used to designate the "f-set of coordinate axes".

NOTE — For a given tool, the set of tool-in-hand axes is unique. The axes are determined only by the operation the tool is "assumed" to perform (for example, for a cylindrical turning tool, the tool-in-hand axes do not change relative to the tool when the tool is used for a facing operation).

2.1.2 Tool-in-use axes

f_e-Set of coordinate axes : X_{fe} (P_{re} P_{pe}), Y_{fe} (P_{pe} P_{fe}), Z_{fe} (P_{fe} P_{re}).

Similarly in this system, the X_{fe}-axis is positive in a direction which is away from both the reference corner at the tool, (as defined in ISO 3002/1) and the assumed machined surface; the Y_{fe}-axis is positive in the direction opposite to the resultant cutting direction; the Z_{fe}-axis is positive in a direction away from the transient surface on the workpiece. [See figures 1 and 2.]

NOTE — The resulting defined axes may conform to either a right hand or a left hand coordinate axis system depending upon the type of tool considered, its intended use and the location of the selected point on the cutting edges.

In a similar way, other sets of tool-in-use axes can be defined. However, for the practical application of the transformation formulae, only the f_e-set of coordinate axes is required. Therefore, in what follows, the more general term "tool-in-use axes" will be used to designate the "f_e-set of coordinate axes".

NOTE — The orientation of the tool-in-use axes, relative to the tool, may vary depending on the resultant cutting direction and the orientation of the tool in the machine.

2.2 Definitions of the machines axes and planes

For each type of machine tool, a machine reference system of axes is defined in ISO 841.

Unfortunately, the defined axis nomenclature is not suitable for direct use in this part of ISO 3002, however, it has been used to determine the orientation of a machine reference system of planes.

With the tool in its "zero position" in the machine such that the tool-in-hand reference system of planes (P_r—P_f—P_p) coincides with the machine reference system of planes (defined by the X—Y—Z axes of ISO 841) and with the tool in its "most natural position", so that the assumed working plane P_f is oriented to be parallel to the direction of component feed motion predominant in the particular machining operation considered, a set of fixed machine setting axes (X_m—Y_m—Z_m) is defined to coincide with the tool-in-hand axes. Thus, with the tool in its "zero position" :

— machine setting axis X_m coincides with tool-in-hand axis X_f;

— machine setting axis Y_m coincides with tool-in-hand axis Y_f;

— machine setting axis Z_m coincides with tool- in-hand axis Z_f.

3 Definition of the setting and motion angles

Two sets of three angles (Euler angles) are required, one set to define the orientation of the tool-in-hand axes (X_f, Y_f, Z_f) in relation to the machine setting axes (X_m, Y_m, Z_m) (see figure 3) and the second set to define the orientation of the tool-in-use axes (X_{fe}, Y_{fe}, Z_{fe}) in relation to the same machine setting axes (see figure 4).

It should be noted that the first set of angles corresponds to the practical procedure of positioning the tool with a classical tool holder [see figure 3].

In some practical cases the second set of angles is not directly related to the available workshop data. For these cases an alternative way of proceeding is outlined in 3.3, using the components of feed speed and cutting speed with respect to the machine setting axes in order to evaluate the auxiliary angles in 4.3.

3.1 The setting angles

The position of the tool-in-hand reference system of planes defined by X_f, Y_f, Z_f, with respect to the machine reference system of planes defined by X_m, Y_m, Z_m is determined by three angles : the plan setting angle G, the elevation setting angle H, the roll setting angle L. The definitions of these angles are given below. The setting angles are illustrated in figures 3 a) and 3 b). It is assumed that the tool is initially in its "zero" position on the machine tool, such that the tool-in-hand reference system of planes coincides with the machine reference system of planes and the tool is in its "most natural" position on the machine tool (see 2.2). The tool is then rotated successively through the angles G, H, L, as illustrated by the graduated scales of the tool holder shown in figure 3 a). The angles G, H, L thus define the positioning of the tool-in-hand reference system of planes with respect to the machine reference system of planes.

The angles may be applied in any order; however, the following explanations of the rotations which are embodied in the definitions assume that the angles are applied in the sequence G, H, L.

3.1.1 The plan setting angle G

The angle between the assumed working plane P_f in its zero position and the assumed working plane P_f in its final position, measured in the tool reference plane P_r in its zero position.

It corresponds to a rotation around the machine setting Y_m-axis. Performing this rotation causes the tool-in-hand coordinate axes to take up an intermediate position in which they are designated X'_f, Y_m, Z'_f. [See figures 3 a) and 3 b).]

The sign convention is defined thus :

Increasing the plan setting angle G in the positive direction decreases the angle κ_{re} and increases the angle ψ_{re}.

3.1.2 The elevation setting angle H

The angle between the machine setting Y_m-axis and its projection on the final position of the assumed working plane P_f of the tool. It corresponds to a rotation around the intermediate Z'_f-axis.

This rotation gives the final position of the tool-in-hand X_f-axis an intermediate position of the tool-in-hand Y_f-axis which is designated Y'_f, and Z'_f is unchanged. Thus, the tool-in-hand axes in this second intermediate position are designated X_f, Y'_f, Z'_f.

The sign convention is defined thus :

Increasing the elevation setting angle H in the positive direction increases the angle γ_{pe} in the positive direction.

3.1.3 The roll setting angle L

The angle between the tool reference plane P_r in its zero position and the tool reference plane P_r in its final position and measured in the assumed working plane P_f in its final position.

It corresponds to a rotation around the final tool-in-hand X_f-axis. This rotation gives the final position of the tool-in-hand axes X_f, Y_f, Z_f in relation to the machine reference axes.

The sign convention is defined thus :

Increasing the roll setting angle L in the positive direction increases the angle α_{fe}.

3.2 The motion angles

The position of the tool-in-use reference system of planes defined by X_{fe}, Y_{fe}, Z_{fe} with respect to the machine reference system of planes defined by X_m, Y_m, Z_m can be determined by three angles : the plan motion angle M, the elevation motion angle N and the roll motion angle T. An alternative method of determining the relative positions is described in 3.3. The definitions of the motion angles are given below. It should be noted that the orientation of the tool-in-use reference system of planes relative to the machine reference system of planes is determined by the direction and magnitudes of cutting speed and feed speed.

The motion angles are illustrated in figure 4. Assume a set of coordinate axes x, y, z, initially coincident with the set of the machine axes X_m, Y_m, Z_m and which can be made coincident with the tool-in-use axes X_{fe}, Y_{fe}, Z_{fe} by means of three Eulerian rotations.

3.2.1 Plan motion angle M

The angle between the $Y_m - Z_m$ machine plane and the working plane P_{fe} measured in the $X_m - Z_m$ machine plane. It corresponds to the auxiliary coordinate system x, y, z being rotated through angle M about the machine setting Y_m-axis into an intermediate position. The coordinate axes in this intermediate position are designated x', Y_m, z' (see figure 4).

The sign convention is defined thus :

Increasing the plan motion angle M in the positive direction increases the angle κ_{re} and decreases the angle ψ_{re}.

In conventional turning, M is the angle between the rotational axis of the workpiece and the direction of feed motion, thus :

- for cylindrical turning $M = 0$ (see figure 5);
- for turning a cone with top angle T : $M = \frac{T}{2}$;
- for most other operations the plan motion angle M is equal to zero.

3.2.2 Elevation motion angle N

The angle between the machine setting Y_m-axis and its projection on the working plane P_{fe}.

It corresponds to the intermediate coordinate system being rotated about the "Z'"-axis from its position x', Y_m, z' into a second intermediate position. The coordinate axes in this second intermediate position are designated X_{fe}, y', z'. This rotation gives the final position of the X_{fe}-axis and an intermediate position of the y'-axis. The designation of z' is unchanged since rotation is about this axis.

The sign convention is defined thus :

Increasing the elevation motion angle N in the positive direction decreases the angle γ_{pe} in the positive direction.

In conventional cylindrical turning, if the selected point on the cutting edge has an offset height h below the rotational axis [see figure 5 b)], the angle N is found from

$$\sin N = \frac{2h}{d}$$

where d is the effective diameter of the workpiece at the selected point on the cutting edge.

In drilling, N is equal to the elevation setting angle H, since for this operation the assumed working plane P_f and the working plane P_{fe} remain coincident and N and H are the angular position of a radius, drawn through the selected point, with respect to the zero position.

In most cases, the elevation motion angle N is zero.

3.2.3 Roll motion angle T

The angle between the $X_m - Z_m$ machine plane and the working reference plane P_{re} measured in the working plane P_{fe}.

It corresponds to a rotation about the X_{fe}-axis of the second intermediate system X_{fe}, y', z' into the tool-in-use system X_{fe}, Y_{fe}, Z_{fe}.

The sign convention is defined thus :

Increasing the roll motion angle T in the positive direction decreases the angle α_{fe}.

In conventional turning (cylindrical, conical, plunge and face turning), if the direction of primary motion is parallel to the machine Y_m-axis, it is the angle between the direction of primary motion and the resultant cutting direction (i.e. resultant cutting speed angle η),

$$\text{tg}\, T = \frac{f}{\pi d}$$

where

d is the effective diameter, in millimetres, of the workpiece at the selected point on the cutting edge;

f is the feed, in millimetres per revolution.

In roll (slab) milling and face milling, it is the sum of the roll setting angle L and the resultant cutting speed angle η (see figure 6).

In drilling, T is equal to the resultant cutting speed angle η.

3.3 Alternative method of relating the tool-in-use reference system of planes to the machine reference system of planes

If the value of the motion angles cannot be deduced easily from available workshop data (i.e. conical turning with a non-centred cutting tool) the position of the tool-in-use reference system of planes defined by X_{fe}, Y_{fe}, Z_{fe} with respect to the machine reference system of planes defined by X_m, Y_m, Z_m can be determined form the X_m, Y_m and Z_m components of the feed speed v_f and cutting speed v_c, which are designated :

components of feed speed $(v_f)_{X_m}$, $(v_f)_{Y_m}$, $(v_f)_{Z_m}$

components of cutting speed $(v_c)_{X_m}$, $(v_c)_{Y_m}$, $(v_c)_{Z_m}$

and used to evaluate auxiliary angles as presented in 4.3.

4 List of conversion formulae — General case

Direct transformation is the expression giving the working angles as a function of the tool angles, setting angles and motion angles. Inverse transformation is the expression giving the tool angles as a function of the working angles, setting angles and motion angles.

In the equations, auxiliary angles are used and are listed in 4.3. They are designated by the general notation (i, j) and are defined as functions of the tool setting angles and the motion angles (respectively components of feed speed and cutting speed).

Although the expressions are complex, in practice they are simplified considerably since certain angles are equal usually to either zero or 90°.

In addition to the conversion formulae below, the table "Relations between the angles in the 'tool-in-hand system' " included in 6.6 of ISO 3002/1 should be used. The same table can be used for the working angles provided that the suffix "e" is added to all angles.

4.1 Direct transformation (working anglers as a function of tool angles)

4.1.1 Working cutting edge inclination

$$\sin \lambda_{se} = \cos \lambda_s \sin \kappa_r \cos (1,2) + \sin \lambda_s \cos (2,2) - \cos \lambda_s \cos \kappa_r \cos (3,2)$$

4.1.2 Working cutting edge angle

$$\text{tg}\, \kappa_{re} = -\left[\frac{\cos \lambda_s \sin \kappa_r \cos (1,1) + \sin \lambda_s \cos (2,1) - \cos \lambda_s \cos \kappa_r \cos (3,1)}{\cos \lambda_s \sin \kappa_r \cos (1,3) + \sin \lambda_s \cos (2,3) - \cos \lambda_s \cos \kappa_r \cos (3,3)}\right]$$

4.1.3 Working normal rake

$$\sin\gamma_{ne} = \frac{1}{\cos\lambda_{se}} \{[-\sin\gamma_n \sin\lambda_s \sin\kappa_r + \cos\gamma_n \cos\kappa_r] \cos(1,2) + [\sin\gamma_n \sin\lambda_s \cos\kappa_r + \cos\gamma_n \sin\kappa_r] \cos(3,2) + \sin\gamma_n \cos\lambda_s \cos(2,2)\}$$

Alternatively :

$$\cos(\gamma_{ne} - \gamma_n) = [\cos\kappa_r \cos(1,1) + \sin\kappa_r \cos(3,1)] \cos\kappa_{re} + [\cos\kappa_r \cos(1,3) + \sin\kappa_r \cos(3,3)] \sin\kappa_{re}$$

NOTE — To make use of this latter relationship it is essential to have previous knowledge of whether the working normal rake is larger or smaller than the tool normal rake.

4.1.4 Working normal clearance

$$\cos\alpha_{ne} = \frac{1}{\cos\lambda_{se}} \{[\sin\alpha_n \cos\kappa_r - \cos\alpha_n \sin\lambda_s \sin\kappa_r] \cos(1,2) + [\cos\alpha_n \sin\lambda_s \cos\kappa_r + \sin\alpha_n \sin\kappa_r] \cos(3,2) + \cos\alpha_n \cos\lambda_s \cos(2,2)\}$$

Alternatively :

$$\cos(\alpha_{ne} - \alpha_n) = [\cos\kappa_r \cos(1,1) + \sin\kappa_r \cos(3,1)] \cos\kappa_{re} + [\cos\kappa_r \cos(1,3) + \sin\kappa_r \cos(3,3)] \sin\kappa_{re}$$

NOTE — To make use of this latter relationship it is essential to have previous knowledge of whether the working normal clearance is larger or smaller than the tool normal clearance.

It should be further noted that if either the working normal rake γ_{ne} or the working normal clearance α_{ne} has been determined, the other can be derived from the relationship :

$$\alpha_{ne} + \beta_{ne} + \gamma_{ne} = 90^\circ \quad \text{and} \quad \beta_{ne} = \beta_n$$

4.1.5 Working side rake

$$\operatorname{tg}\gamma_{fe} = \frac{\operatorname{tg}\gamma_f \cos(3,3) + \operatorname{tg}\gamma_p \cos(1,3) - \cos(2,3)}{\cos(2,2) - \operatorname{tg}\gamma_f \cos(3,2) - \operatorname{tg}\gamma_p \cos(1,2)}$$

4.1.6 Working back rake

$$\operatorname{tg}\gamma_{pe} = \frac{\operatorname{tg}\gamma_f \cos(3,1) + \operatorname{tg}\gamma_p \cos(1,1) - \cos(2,1)}{\cos(2,2) - \operatorname{tg}\gamma_f \cos(3,2) - \operatorname{tg}\gamma_p \cos(1,2)}$$

4.1.7 Working side clearance

$$\operatorname{ctg}\alpha_{fe} = \frac{\operatorname{ctg}\alpha_f \cos(3,3) + \operatorname{ctg}\alpha_p \cos(1,3) - \cos(2,3)}{\cos(2,2) - \operatorname{ctg}\alpha_f \cos(3,2) - \operatorname{ctg}\alpha_p \cos(1,2)}$$

4.1.8 Working back clearance

$$\operatorname{ctg}\alpha_{pe} = \frac{\operatorname{ctg}\alpha_f \cos(3,1) + \operatorname{ctg}\alpha_p \cos(1,1) - \cos(2,1)}{\cos(2,2) - \operatorname{ctg}\alpha_f \cos(3,2) - \operatorname{ctg}\alpha_p \cos(1,2)}$$

4.2 Inverse transformation (tool angles as a function of working angles)

4.2.1 Tool cutting edge inclination

$$\sin\lambda_s = \cos\lambda_{se} \sin\kappa_{re} \cos(2,1) + \sin\lambda_{se} \cos(2,2) - \cos\lambda_{se} \cos\kappa_{re} \cos(2,3)$$

4.2.2 Tool cutting edge angle

$$\operatorname{tg}\kappa_r = -\left[\frac{\cos\lambda_{se} \sin\kappa_{re} \cos(1,1) + \sin\lambda_{se} \cos(1,2) - \cos\lambda_{se} \cos\kappa_{re} \cos(1,3)}{\cos\lambda_{se} \sin\kappa_{re} \cos(3,1) + \sin\lambda_{se} \cos(3,2) - \cos\lambda_{se} \cos\kappa_{re} \cos(3,3)}\right]$$

4.2.3 Tool normal rake

$$\sin \gamma_n = \frac{1}{\cos \lambda_s} \{ [-\sin \gamma_{ne} \sin \lambda_{se} \sin \kappa_{re} + \cos \gamma_{ne} \cos \kappa_{re}] \cos (2,1) + [\sin \gamma_{ne} \sin \lambda_{se} \cos \kappa_{re} + \cos \gamma_{ne} \sin \kappa_{re}] \cos (2,3) + \sin \gamma_{ne} \cos \lambda_{se} \cos (2,2) \}$$

Alternatively, the tool normal rake γ_n may be derived from the expression for cos $(\gamma_{ne} - \gamma_n)$ which is listed in 4.1. However, to use this relationship it is essential to have previous knowledge of whether the tool normal rake is larger or smaller than the working normal rake.

4.2.4 Tool normal clearance

$$\cos \alpha_n = \frac{1}{\cos \lambda_s} \{ [-\cos \alpha_{ne} \sin \lambda_{se} \sin \kappa_{re} + \sin \alpha_{ne} \cos \kappa_{re}] \cos (2,1) + [\cos \alpha_{ne} \sin \lambda_{se} \cos \kappa_{re} + \sin \alpha_{ne} \sin \kappa_{re}] \cos (2,3) + \cos \alpha_{ne} \cos \lambda_{se} \cos (2,2) \}$$

Alternatively, the tool normal clearance α_n may be derived from the expression for cos $(\alpha_{ne} - \alpha_n)$ which is listed in 4.1. However, to use this relationship it is essential to have previous knowledge of whether the tool normal clearance is larger or smaller than the working normal clearance.

It should be further noted that if either the tool normal rake or tool normal clearance has been determined, the other can be derived from the relationship :

$$\alpha_n + \beta_n + \gamma_n = 90^\circ \quad \text{and} \quad \beta_n = \beta_{ne}$$

4.2.5 Tool side rake

$$\operatorname{tg} \gamma_f = \frac{\operatorname{tg} \gamma_{fe} \cos (3,3) + \operatorname{tg} \gamma_{pe} \cos (3,1) - \cos (3,2)}{-\operatorname{tg} \gamma_{fe} \cos (2,3) - \operatorname{tg} \gamma_{pe} \cos (2,1) + \cos (2,2)}$$

4.2.6 Tool back rake

$$\operatorname{tg} \gamma_p = \frac{\operatorname{tg} \gamma_{fe} \cos (1,3) + \operatorname{tg} \gamma_{pe} \cos (1,1) - \cos (1,2)}{-\operatorname{tg} \gamma_{fe} \cos (2,3) - \operatorname{tg} \gamma_{pe} \cos (2,1) + \cos (2,2)}$$

4.2.7 Tool side clearance

$$\operatorname{ctg} \alpha_f = \frac{\operatorname{ctg} \alpha_{fe} \cos (3,3) + \operatorname{ctg} \alpha_{pe} \cos (3,1) - \cos (3,2)}{-\operatorname{ctg} \alpha_{fe} \cos (2,3) - \operatorname{ctg} \alpha_{pe} \cos (2,1) + \cos (2,2)}$$

4.2.8 Tool back clearance

$$\operatorname{ctg} \alpha_p = \frac{\operatorname{ctg} \alpha_{fe} \cos (1,3) + \operatorname{ctg} \alpha_{pe} \cos (1,1) - \cos (1,2)}{-\operatorname{ctg} \alpha_{fe} \cos (2,3) - \operatorname{ctg} \alpha_{pe} \cos (2,1) + \cos (2,2)}$$

4.3 Auxiliary angles

The geometrical meaning of the auxiliary angles (i, j) is as follows :

The angle (1,1) is the angle between the axes X_f and X_{fe}

The angle (2,1) is the angle between the axes Y_f and X_{fe}

The angle (3,1) is the angle between the axes Z_f and X_{fe}

The angle (1,2) is the angle between the axes X_f and Y_{fe}

The angle (2,2) is the angle between the axes Y_f and Y_{fe}

The angle (3,2) is the angle between the axes Z_f and Y_{fe}

The angle (1,3) is the angle between the axes X_f and Z_{fe}

The angle (2,3) is the angle between the axes Y_f and Z_{fe}

The angle (3,3) is the angle between the axes Z_f and Z_{fe}

The values of cos (i, j) can be expressed as a combination of the cosines of the angles between the tool-in-hand axes (X_f, Y_f, Z_f) and the machine setting axes (X_m, Y_m, Z_m) on one hand and the cosines of the angles between the tool-in-use axes (X_{fe}, Y_{fe}, Z_{fe}) and the machine setting axes (X_m, Y_m, Z_m) on the other hand :

$$\cos(1,1) = \cos(X_f, X_m)\cos(X_{fe}, X_m) + \cos(X_f, Y_m)\cos(X_{fe}, Y_m) + \cos(X_f, Z_m)\cos(X_{fe}, Z_m)$$

$$\cos(2,1) = \cos(Y_f, X_m)\cos(X_{fe}, X_m) + \cos(Y_f, Y_m)\cos(X_{fe}, Y_m) + \cos(Y_f, Z_m)\cos(X_{fe}, Z_m)$$

$$\cos(3,1) = \cos(Z_f, X_m)\cos(X_{fe}, X_m) + \cos(Z_f, Y_m)\cos(X_{fe}, Y_m) + \cos(Z_f, Z_m)\cos(X_{fe}, Z_m)$$

$$\cos(1,2) = \cos(X_f, X_m)\cos(Y_{fe}, X_m) + \cos(X_f, Y_m)\cos(Y_{fe}, Y_m) + \cos(X_f, Z_m)\cos(Y_{fe}, Z_m)$$

$$\cos(2,2) = \cos(Y_f, X_m)\cos(Y_{fe}, X_m) + \cos(Y_f, Y_m)\cos(Y_{fe}, Y_m) + \cos(Y_f, Z_m)\cos(Y_{fe}, Z_m)$$

$$\cos(3,2) = \cos(Z_f, X_m)\cos(Y_{fe}, X_m) + \cos(Z_f, Y_m)\cos(Y_{fe}, Y_m) + \cos(Z_f, Z_m)\cos(Y_{fe}, Z_m)$$

$$\cos(1,3) = \cos(X_f, X_m)\cos(Z_{fe}, X_m) + \cos(X_f, Y_m)\cos(Z_{fe}, Y_m) + \cos(X_f, Z_m)\cos(Z_{fe}, Z_m)$$

$$\cos(2,3) = \cos(Y_f, X_m)\cos(Z_{fe}, X_m) + \cos(Y_f, Y_m)\cos(Z_{fe}, Y_m) + \cos(Y_f, Z_m)\cos(Z_{fe}, Z_m)$$

$$\cos(3,3) = \cos(Z_f, X_m)\cos(Z_{fe}, X_m) + \cos(Z_f, Y_m)\cos(Z_{fe}, Y_m) + \cos(Z_f, Z_m)\cos(Z_{fe}, Z_m)$$

The angles between the tool-in-hand axes and machine setting axes can be expressed as a function of the setting angles :

$$\cos(X_f, X_m) = \cos G \cos H$$

$$\cos(X_f, Y_m) = \sin H$$

$$\cos(X_f, Z_m) = -\sin G \cos H$$

$$\cos(Y_f, X_m) = -\cos G \sin H \cos L + \sin G \sin L$$

$$\cos(Y_f, Y_m) = \cos H \cos L$$

$$\cos(Y_f, Z_m) = \sin G \sin H \cos L + \cos G \sin L$$

$$\cos(Z_f, X_m) = \cos G \sin H \sin L + \sin G \cos L$$

$$\cos(Z_f, Y_m) = -\cos H \sin L$$

$$\cos(Z_f, Z_m) = -\sin G \sin H \sin L + \cos G \cos L$$

The angles between the tool-in-use axes and the machine setting axes can be expressed in two different ways : as a function of the motion angles (1st way) or as a function of the components of cutting speed and feed speed (2nd way).

1st way

$$\cos(X_{fe}, X_m) = \cos M \cos N$$

$$\cos(X_{fe}, Y_m) = \sin N$$

$$\cos(X_{fe}, Z_m) = -\sin M \cos N$$

$$\cos(Y_{fe}, X_m) = -\cos M \sin N \cos T + \sin M \sin T$$

$$\cos(Y_{fe}, Y_m) = \cos N \cos T$$

$$\cos(Y_{fe}, Z_m) = \sin M \sin N \cos T + \cos M \sin T$$

$$\cos(Z_{fe}, X_m) = \cos M \sin N \sin T + \sin M \cos T$$

$$\cos(Z_{fe}, Y_m) = -\cos N \sin T$$

$$\cos(Z_{fe}, Z_m) = -\sin M \sin N \sin T + \cos M \cos T$$

2nd way

$$\cos(X_{fe}, X_m) = \frac{(v_c)_{Y_m}(v_f)_{Z_m} - (v_c)_{Z_m}(v_f)_{Y_m}}{|\sin\varphi\, v_c\, v_f|}$$

$$\cos(X_{fe}, Y_m) = \frac{(v_c)_{Z_m}(v_f)_{X_m} - (v_c)_{X_m}(v_f)_{Z_m}}{|\sin\varphi\, v_c\, v_f|}$$

$$\cos(X_{fe}, Z_m) = \frac{(v_c)_{X_m}(v_f)_{Y_m} - (v_c)_{Y_m}(v_f)_{X_m}}{|\sin\varphi\, v_c\, v_f|}$$

$$\cos(Y_{fe}, X_m) = -\frac{(v_c)_{X_m} + (v_f)_{X_m}}{|v_e|}$$

$$\cos(Y_{fe}, Y_m) = -\frac{(v_c)_{Y_m} + (v_f)_{Y_m}}{|v_e|}$$

$$\cos(Y_{fe}, Z_m) = -\frac{(v_c)_{Z_m} + (v_f)_{Z_m}}{|v_e|}$$

$$\cos(Z_{fe}, X_m) = \cos(X_{fe}, Y_m)\cos(Y_{fe}, Z_m) - \cos(X_{fe}, Z_m)\cos(Y_{fe}, Y_m)$$

$$\cos(Z_{fe}, Y_m) = \cos(X_{fe}, Z_m)\cos(Y_{fe}, X_m) - \cos(X_{fe}, X_m)\cos(Y_{fe}, Z_m)$$

$$\cos(Z_{fe}, Z_m) = \cos(X_{fe}, X_m)\cos(Y_{fe}, Y_m) - \cos(X_{fe}, Y_m)\cos(Y_{fe}, X_m)$$

The product $\sin\varphi\, v_c\, v_f$ can either be computed directly or can be derived from the following expression :

$$|\sin\varphi\, v_c\, v_f| = \sqrt{\left[(v_c)_{Y_m}(v_f)_{Z_m} - (v_c)_{Z_m}(v_f)_{Y_m}\right]^2 + \left[(v_c)_{Z_m}(v_f)_{X_m} - (v_c)_{X_m}(v_f)_{Z_m}\right]^2 + \left[(v_c)_{X_m}(v_f)_{Y_m} - (v_c)_{Y_m}(v_f)_{X_m}\right]^2}$$

where

φ is the feed motion angle;

v_c is the magnitude of the cutting speed;

v_f is the magnitude of the feed speed.

NOTE — In certain cases, depending upon the type of tool considered, its intended use and the location of the selected point on the cutting edge (for example, for a selected point located on the major cutting edge of a L.H. turning tool or for a selected point located on the minor cutting edge of a R.H. turning tool), the value of the term $[(v_c)_{Y_m}(v_f)_{Z_m} - (v_c)_{Z_m}(v_f)_{Y_m}]$ may be negative.

In such cases $|\sin\varphi\, v_c\, v_f|$ should be replaced by $-|\sin\varphi\, v_c\, v_f|$ in all formulae.

The resulting cutting speed v_e can be derived from

$$|v_e| = \sqrt{\left[(v_c)_{X_m} + (v_f)_{X_m}\right]^2 + \left[(v_c)_{Y_m} + (v_f)_{Y_m}\right]^2 + \left[(v_c)_{Z_m} + (v_f)_{Z_m}\right]^2}$$

5 Conversion formulae for cases where the working plane P_{fe} coincide with the assumed working plane P_f

5.1 Applications

The types of cutting operations in which the working plane P_{fe} and the assumed working plane P_f are coincident are :

- all drilling and similar processes;
- most milling operations;
- most plunge turning operations and certain conventional cylindrical turning operations.

If the plane P_{fe} coincides with the plane P_f, the setting and motion angles have the following relationships : the plan setting angle G may have an arbitrary non-zero value, but it is always indentical to the plan motion angle M, i.e.

$$G \equiv M \qquad \ldots (1)$$

The same is true for the elevation setting angle H and the elevation motion angle N, i.e.

$$H \equiv N \qquad \ldots (2)$$

The roll setting angle L and the roll motion angle T are arbitrary and are not directly related.

From relationships (1) and (2) above, the evaluation of the auxiliary angles listed in 4.3 gives :

$\cos(1,1) = 1$	$\cos(1,2) = 0$	$\cos(1,3) = 0$
$\cos(2,1) = 0$	$\cos(2,2) = \cos(T-L)$	$\cos(2,3) = -\sin(T-L)$
$\cos(3,1) = 0$	$\cos(3,2) = \sin(T-L)$	$\cos(3,3) = \cos(T-L)$

In the machining operations listed above, the roll setting angle L and the roll motion angle T are the only ones required for the conversions. The angle $(T-L)$ is the angle between the assumed direction of primary motion (Y_f-axis) and the resultant cutting direction (Y_{fe}-axis).

NOTE — From the expressions of the auxiliary angles and the physical meaning of the angle $(T-L)$, it follows that the tool-in-hand axes and tool-in-use axes are directly related to each other without using the machine setting axes. Therefore, the "zero position" of the tool on the machine need not be defined for the machining operations listed above.

5.2 Direct transformation

By substituting the above values for the auxiliary angles in the general conversion formulae, we have :

$$\sin\lambda_{se} = \sin\lambda_s \cos(T-L) - \cos\lambda_s \cos\kappa_r \sin(T-L)$$

$$\operatorname{tg}\kappa_{re} = \frac{\cos\lambda_s \sin\kappa_r}{\sin\lambda_s \sin(T-L) + \cos\lambda_s \cos\kappa_r \cos(T-L)}$$

$$\sin\gamma_{ne} = \frac{1}{\cos\lambda_{se}}\left[(\sin\gamma_n \sin\lambda_s \cos\kappa_r + \cos\gamma_n \sin\kappa_r)\sin(T-L) + \sin\gamma_n \cos\lambda_s \cos(T-L)\right]$$

5.3 Inverse transformation

Again by substituting the above values for the auxiliary angles, we have :

$$\sin\lambda_s = \sin\lambda_{se}\cos(T-L) + \cos\lambda_{se}\cos\kappa_{re}\sin(T-L)$$

$$\operatorname{tg}\kappa_r = \frac{-\cos\lambda_{se}\sin\kappa_{re}}{\sin\lambda_{se}\sin(T-L) - \cos\lambda_{se}\cos\kappa_{re}\cos(T-L)}$$

$$\sin\gamma_n = \frac{1}{\cos\lambda_s}\left[-(\sin\gamma_{ne}\sin\lambda_{se}\cos\kappa_{re} + \cos\gamma_{ne}\sin\kappa_{re})\sin(T-L) + \sin\gamma_{ne}\cos\lambda_{se}\cos(T-L)\right]$$

6 Practical examples

6.1 Cylindrical turning (see figure 5)

In general, the tool reference plane P_r will remain in its zero position; so $H = L = 0$. The plan setting angle G may not, however, be zero. For cylindrical turning, the direction of feed motion coincides with the machine setting Z_m-axis.

6.1.1 When the selected point on the cutting edge is centred at the level of the rotation axis the following relationships result [see figure 5 a)] :

The angle N is equal to zero.

The angle T can be determined by

$$\operatorname{tg} T = \frac{f}{\pi d}$$

where

d is the effective diameter, in millimetres, of the workpiece at the selected point on the cutting edge;

f is the feed, in millimetres per revolution.

The auxiliary angles have the following values :

$\cos(1,1) = \cos G$ $\quad \cos(1,2) = -\sin T \sin G$ $\quad \cos(1,3) = -\cos T \sin G$

$\cos(2,1) = 0$ $\quad \cos(2,2) = \cos T$ $\quad \cos(2,3) = -\sin T$

$\cos(3,1) = \sin G$ $\quad \cos(3,2) = \cos G \sin T$ $\quad \cos(3,3) = \cos G \cos T$

The resultant conversion formulae are :

6.1.1.1 Direct transformation

$$\sin\lambda_{se} = \sin\lambda_s \cos T - \cos\lambda_s \sin T \cos(\kappa_r - G)$$

$$\mathrm{tg}\,\kappa_{re} = \frac{\cos\lambda_s \sin(\kappa_r - G)}{\cos\lambda_s \cos T \cos(\kappa_r - G) + \sin\lambda_s \sin T}$$

$$\sin\gamma_{ne} = \frac{1}{\cos\lambda_{se}}\left[\cos(\kappa_r - G)\sin T \sin\lambda_s \sin\gamma_n + \cos T \sin\gamma_n \cos\lambda_s + \sin(\kappa_r - G)\cos\gamma_n \sin T\right]$$

6.1.1.2 Inverse transformation

$$\sin\lambda_s = \sin\lambda_{se}\cos T + \cos\lambda_{se}\cos\kappa_{re}\sin T$$

$$\mathrm{tg}\,\kappa_r = \frac{-\cos\lambda_{se}\sin\kappa_{re}\cos G + \sin\lambda_{se}\sin T\sin G - \cos\lambda_{se}\cos\kappa_{re}\cos T\sin G}{\cos\lambda_{se}\sin\kappa_{re}\sin G + \sin\lambda_{se}\cos G\sin T - \cos\lambda_{se}\cos\kappa_{re}\cos G\cos T}$$

$$\sin\gamma_n = \frac{1}{\cos\lambda_s}\left\{\sin\gamma_{ne}\cos\lambda_{se}\cos T - [\sin\gamma_{ne}\sin\lambda_{se}\cos\kappa_{re} + \cos\gamma_{ne}\sin\kappa_{re}]\sin T\right\}$$

NOTE — Using the 2nd way of describing the relationship between the tool-in-use system and the machine setting system, the auxiliary angles have the following expressions [with the exception of components $(v_c)_{Y_m}$ and $(v_f)_{Z_m}$, all other components are equal to zero for a cylindrical turning operation] :

$\cos(1,1) = \cos G$

$\cos(1,2) = \sin G \dfrac{(v_f)_{Z_m}}{|v_e|}$

$\cos(1,3) = \sin G \dfrac{(v_f)_{Y_m}}{|v_e|}$

$\cos(2,1) = 0$

$\cos(2,2) = -\dfrac{(v_c)_{Y_m}}{|v_e|}$

$\cos(2,3) = \dfrac{(v_f)_{Z_m}}{|v_e|}$

$\cos(3,1) = \sin G$

$\cos(3,2) = -\cos G \dfrac{(v_f)_{Z_m}}{|v_e|}$

$\cos(3,3) = -\cos G \dfrac{(v_c)_{Y_m}}{|v_e|}$

Since

$\sin T = -\dfrac{(v_f)_{Z_m}}{|v_e|}$ and $\cos T = -\dfrac{(v_c)_{Y_m}}{|v_e|}$

these values correspond to the ones given above.

6.1.2 When the selected point on the cutting edge has an offset distance h in relation to the rotation axis, the following relationships result [see figure 5 b)] :

The angle N can be found by

$$\sin N = \frac{2h}{d}$$

where d is the effective diameter of the workpiece at the selected point on the cutting edge.

The angle T has the same value as above.

The auxiliary angles have the following values :

$$\cos(1,1) = \cos N \cos G$$

$$\cos(2,1) = \sin N$$

$$\cos(3,1) = \cos N \sin G$$

$$\cos(1,2) = -\sin N \cos T \cos G - \sin T \sin G$$

$$\cos(2,2) = \cos N \cos T$$

$$\cos(3,2) = \cos G \sin T - \sin G \sin N \cos T$$

$$\cos(1,3) = \sin N \sin T \cos G - \cos T \sin G$$

$$\cos(2,3) = -\cos N \sin T$$

$$\cos(3,3) = \cos G \cos T + \sin G \sin N \sin T$$

The resultant formulae are :

6.1.2.1 Direct transformation

$$\sin \lambda_{se} = -\cos \lambda_s \sin N \cos T \sin(\kappa_r - G) - \cos \lambda_s \sin T \cos(\kappa_r - G) + \sin \lambda_s \cos N \cos T$$

$$\mathrm{tg}\, \kappa_{re} = \frac{\cos \lambda_s \cos N \sin(\kappa_r - G) + \sin \lambda_s \sin N}{-\cos \lambda_s \sin N \sin T \sin(\kappa_r - G) + \cos \lambda_s \cos T \cos(\kappa_r - G) + \sin \lambda_s \cos N \sin T}$$

$$\sin \gamma_{ne} = \frac{1}{\cos \lambda_{se}} [\sin \gamma_n \sin \lambda_s \sin N \cos T \sin(\kappa_r - G) + \sin \gamma_n \sin \lambda_s \sin T \cos(\kappa_r - G) - \cos \gamma_n \sin N \cos T \cos(\kappa_r - G) + \cos \gamma_n \sin T \sin(\kappa_r - G) + \sin \gamma_n \cos \lambda_s \cos N \cos T]$$

6.1.2.2 Inverse transformation

$$\sin\lambda_s = \cos\lambda_{se}\sin\kappa_{re}\sin N + \sin\lambda_{se}\cos N\cos T + \cos\lambda_{se}\cos\kappa_{re}\cos N\sin T$$

$$\operatorname{tg}\kappa_r = \frac{-\cos\lambda_{se}\sin\kappa_{re}\cos N\cos G + \sin\lambda_{se}[\sin N\cos T\cos G + \sin T\sin G] + \cos\lambda_{se}\cos\kappa_{re}[\sin N\sin T\cos G - \cos T\sin G]}{\cos\lambda_{se}\sin\kappa_{re}\cos N\sin G + \sin\lambda_{se}[\cos G\sin T - \sin G\sin N\cos T] - \cos\lambda_{se}\cos\kappa_{re}[\cos G\cos T + \sin G\sin N\sin T]}$$

$$\sin\gamma_n = \frac{1}{\cos\lambda_s}\Big\{[-\sin\gamma_{ne}\sin\lambda_{se}\sin\kappa_{re} + \cos\gamma_{ne}\cos\kappa_{re}]\sin N + \sin\gamma_{ne}\cos\lambda_{se}\cos N\cos T -$$

$$-[\sin\gamma_{ne}\sin\lambda_{se}\cos\kappa_{re} + \cos\gamma_{ne}\sin\kappa_{re}]\cos N\sin T\Big\}$$

NOTE — Using the 2nd way of describing the relationship between the tool-in-use system and the machine setting system, the auxiliary angles have the following expressions [only the components $(v_c)_{Y_m}$, $(v_c)_{X_m}$ and $(v_f)_{Z_m}$ are not equal to zero for a cylindrical operation] :

$$\cos(1,1) = -\cos G\,\frac{(v_c)_{Y_m}}{|v_c|}$$

$$\cos(2,1) = \frac{(v_c)_{X_m}}{|v_c|}$$

$$\cos(3,1) = -\sin G\,\frac{(v_c)_{Y_m}}{|v_c|}$$

$$\cos(1,2) = -\cos G\,\frac{(v_c)_{X_m}}{|v_e|} + \sin G\,\frac{(v_f)_{Z_m}}{|v_e|}$$

$$\cos(2,2) = -\frac{(v_c)_{Y_m}}{|v_e|}$$

$$\cos(3,2) = -\sin G\,\frac{(v_c)_{X_m}}{|v_e|} - \cos G\,\frac{(v_f)_{Z_m}}{|v_e|}$$

$$\cos(1,3) = -\cos G\,\frac{(v_c)_{X_m}\,(v_f)_{Z_m}}{|v_c|\,|v_e|} - \sin G\,\frac{|v_c|}{|v_e|}$$

$$\cos(2,3) = -\frac{(v_c)_{Y_m}\,(v_f)_{Z_m}}{|v_c|\,|v_e|}$$

$$\cos(3,3) = -\sin G\,\frac{(v_c)_{X_m}\,(v_f)_{Z_m}}{|v_c|\,|v_e|} + \cos G\,\frac{|v_c|}{|v_e|}$$

It can be verified that these values correspond to the ones given above.

6.2 Conical turning

The selected point on the cutting edge is supposed to have an offset distance h in relation to the rotation axes. Suppose that $H = L = 0$ and $G \neq 0$.

In this case, the motion angles cannot be computed easily from the available cutting data. The second way of describing the relationship between the tool-in-use system and the machine cutting system is used.

The components of feed and cutting speed, not equal to zero, are :

$(v_f)_{X_m} \quad (v_f)_{Z_m} \quad (v_c)_{X_m} \quad (v_c)_{Y_m}$

The following values can be computed :

$$|\sin \varphi\, v_c\, v_f| = \sqrt{(v_c)^2_{Y_m}\,(v_f)^2_{Z_m} + (v_c)^2_{X_m}\,(v_f)^2_{Z_m} + (v_c)^2_{Y_m}\,(v_f)^2_{X_m}}$$

$$|v_e| = \sqrt{\left[(v_c)_{X_m} + (v_f)_{X_m}\right]^2 + (v_c)^2_{Y_m} + (v_f)^2_{Z_m}}$$

The auxiliary angles have the following values :

$$\cos(1,1) = \frac{\cos G\,(v_c)_{Y_m}\,(v_f)_{Z_m} + \sin G\,(v_c)_{Y_m}\,(v_f)_{X_m}}{|\sin \varphi\, v_c\, v_f|}$$

$$\cos(2,1) = \frac{-(v_c)_{X_m}\,(v_f)_{Z_m}}{|\sin \varphi\, v_c\, v_f|}$$

$$\cos(3,1) = \frac{\sin G\,(v_c)_{Y_m}\,(v_f)_{Z_m} - \cos G\,(v_c)_{Y_m}\,(v_f)_{X_m}}{|\sin \varphi\, v_c\, v_f|}$$

$$\cos(1,2) = \frac{-\cos G\left[(v_c)_{X_m} + (v_f)_{X_m}\right] + \sin G\,(v_f)_{Z_m}}{|v_e|}$$

$$\cos(2,2) = \frac{-(v_c)_{Y_m}}{|v_e|}$$

$$\cos(3,2) = \frac{-\sin G\left[(v_c)_{X_m} + (v_f)_{X_m}\right] - \cos G\,(v_f)_{Z_m}}{|v_e|}$$

$$\cos(1,3) = \frac{\cos G\left[(v_c)_{X_m}\,(v_f)^2_{Z_m} - (v_c)^2_{Y_m}\,(v_f)_{X_m}\right] + \sin G\,(v_f)_{Z_m}\left[|v_c|^2 + (v_c)_{X_m}\,(v_f)_{X_m}\right]}{|\sin \varphi\, v_c\, v_f|\;|v_e|}$$

$$\cos(2,3) = \frac{(v_c)_{Y_m}\left[(v_c)_{X_m}\,(v_f)_{X_m} + |v_f|^2\right]}{|\sin \varphi\, v_c\, v_f|\;|v_e|}$$

$$\cos(3,3) = \frac{\sin G\left[(v_c)_{X_m}\,(v_f)^2_{Z_m} - (v_c)^2_{Y_m}\,(v_f)_{X_m}\right] - \cos G\,(v_f)_{Z_m}\left[|v_c|^2 + (v_c)_{X_m}\,(v_f)_{X_m}\right]}{|\sin \varphi\, v_c\, v_f|\;|v_e|}$$

Special attention should be paid to the signs of the components of the feed speed and cutting speed.

NOTE — In certain cases, depending upon the type of tool considered, its intended use and the location of the selected point on the cutting edge (for example, for a selected point located on the major cutting edge of a L.H. turning tool or for a selected point located on the minor cutting edge of a R.H. turning tool), the value of the term $[(v_c)_{Y_m}\,(v_f)_{Z_m} - (v_c)_{Z_m}\,(v_f)_{Y_m}]$ may be negative.

In such cases $|\sin \varphi\, v_c\, v_f|$ should be replaced by $-|\sin \varphi\, v_c\, v_f|$ in all formulae.

The following situation generally occurs :

- $(v_c)_{Y_m}$ and $(v_c)_{Z_m}$ are negative;
- if the selected point on the cutting edge is below the rotation axis, then $(v_c)_{X_m}$ is positive;
- if the cone to be turned has an increasing diameter in the direction of feed motion, then $(v_f)_{X_m}$ is positive.

6.3 Roll (slab) milling (see figure 6)

If a helical milling cutter is considered, then :

a) the tool cutting edge inclination λ_s = the helix angle;

b) the tool cutting edge angle $\kappa_r = 90°$;

c) the angle $(T-L)$ = the resultant cutting speed angle η, since the direction of primary motion coincides with the assumed direction of primary motion;

d) the planes P_f and P_{fe} are coincident.

Substituting the above conditions in the conversion formulae developed in clause 5, we have :

6.3.1 Direct transformation

$$\sin \lambda_{se} = \sin \lambda_s \cos \eta$$

$$\operatorname{tg} \kappa_{re} = \frac{\cos \lambda_s}{\sin \lambda_s \sin \eta}$$

$$\sin \gamma_{ne} = \frac{1}{\cos \lambda_{se}} (\cos \gamma_n \sin \eta + \sin \gamma_n \cos \lambda_s \cos \eta)$$

6.3.2 Inverse transformation

$$\sin \lambda_s = \sin \lambda_{se} \cos \eta + \cos \lambda_{se} \cos \kappa_{re} \sin \eta$$

$$\sin \gamma_n = \frac{1}{\cos \lambda_s} [-(\sin \gamma_{ne} \sin \lambda_{se} \cos \kappa_{re} + \cos \gamma_{ne} \sin \kappa_{re}) \sin \eta + \sin \gamma_{ne} \cos \lambda_{se} \cos \eta]$$

NOTE — As explained in 5.1, the "zero position" of the tool on the machine need not be defined for this operation.

Annex A

Elaboration of conversion formulae

Many methods including the geometric projection method, the trigonometric method and the matrix transformation method can be used for deriving the general conversion formulae given in this part of ISO 3002. The matrix method has been used because it is particularly suited for computerized application.

A.1 General transformation scheme

The procedure which is followed to establish the relationships between the tool-in-hand system and the tool-in-use system can be explained briefly as follows :

Initially, a system of coordinate axes is established for the tool in relation to the cutting edge and this is called the 0-set of coordinate axes. The axes in this system are designated X_0, Y_0, Z_0. Figure 7 illustrates these axes, which are defined by the intersection of tool-in-hand planes as follows :

X_0 = line of intersection of the tool cutting edge plane P_s and the tool reference plane P_r.

Y_0 = line of intersection of the tool orthogonal plane P_o and the tool cutting edge plane P_s.

Z_0 = line of intersection of the tool orthogonal plane P_o and the tool reference plane P_r.

Within this coordinate axis system two characteristic lines are considered, firstly the tool cutting edge and secondly the line of intersection of the cutting edge normal plane P_n and the face. The orientation of these lines within the coordinate axis system is determined by the tool cutting edge inclination λ_s and the tool normal rake γ_n (see figure 7).

The direction cosines of the lines can be written :

cutting edge : $(\cos\lambda_s, \sin\lambda_s, 0)$

intersection of P_n and face : $(-\sin\gamma_n \sin\lambda_s, \sin\gamma_n \cos\lambda_s, \cos\gamma_n)$

These direction cosines are combined in the matrix G :

$$G = \begin{vmatrix} \cos\lambda_s & \sin\lambda_s & 0 \\ -\sin\gamma_n \sin\lambda_s & \sin\gamma_n \cos\lambda_s & \cos\gamma_n \end{vmatrix}$$

If now a second system of axes, the 0_e-set of coordinate axes, is considered and defined in relation to the working cutting edge plane P_{se}, the working reference plane P_{re} and the working orthogonal plane P_{oe} and the same lines considered within this coordinate system, it follows that :

$$G_e = \begin{vmatrix} \cos\lambda_{se} & \sin\lambda_{se} & 0 \\ -\sin\gamma_{ne} \sin\lambda_{se} & \sin\gamma_{ne} \cos\lambda_{se} & \cos\gamma_{ne} \end{vmatrix}$$

Figure 8 a) outlines the transformation order. The matrix G which defines the two characteristic lines of the tool in the 0-set of coordinate axes is transformed through the f-set of coordinate axes and machine reference system into the f_e-set of coordinate axes. Similarly, the matrix G_e which defines the same two characteristic lines of the tool in the 0_e-set of coordinate axes is transformed into f_e-set of coordinate axes.

Thus, two different expressions for the direction cosines of the same lines in the same reference axis system are obtained and by equating the corresponding elements, the required relationships are obtained.

Each transformation step consists of a rotation of the considered coordinate axis system about one of its axes. Tables 1 and 2 summarize the rotations performed, the axis about which rotation occurs, the original coordinate axis designation and the resulting axis designation :

Table 1 — Transformation from the 0-set of coordinate exes to the f_e-set of coordinate axes

Rotation angle	Rotation axis	Actual reference system	Reference system after transformation
$90° - \kappa_r$	Y_0	0-Set of coordinate axes X_0, Y_0, Z_0	f-Set of coordinate axes X_f, Y_f, Z_f (final position)
$-L$	X_f	f-Set of coordinate axes X_f, Y_f, Z_f	Intermediate system 2
$-H$	Z'_f	Intermediate system 2	Intermediate system 1
$-G$	Y_m	Intermediate system 1	Machine setting system X_m, Y_m, Z_m
M	Y_m	Machine setting system X_m, Y_m, Z_m	Intermediate system 1
N	z'	Intermediate system 1	Intermediate system 2
T	X_{fe}	Intermediate system 2	f_e-Set of coordinate axes X_{fe}, Y_{fe}, Z_{fe}

Table 2 — Transformation from the 0_e-set of coordinate axes to the f_e-set of coordinate axes

Rotation angle	Rotation axis	Actual reference system	Reference system after transformation
$90° - \kappa_r$	Y_{0e}	0_e-Set of coordinate axes X_{0e}, Y_{0e}, Z_{0e}	f_e-Set of coordinate axes X_{fe}, Y_{fe}, Z_{fe}

A.2 Details of the transformations

Each of the transformation steps described above is characterized by a transformation matrix of order 3.

The general method for setting up an elementary transformation matrix is given in annex B.

In the following paragraphs, the transformation of the G-matrix and of the G_e-matrix into the f_e-set of coordinate axes will be discussed.

A.2.1 Transformation from the 0-set of coordinate axes to the f_e-set of coordinate axes

A.2.1.1 Transformation from the 0-set of coordinate axes to the f-set of coordinate axes transformation K_r)

The reference systems have their Y-axes in common. It is therefore obvious that one system can coincide with the other by rotation around its Y-axis. From the definition of the tool cutting edge angle κ_r (angle between the tool cutting edge plane P_s and the assumed working plane P_f), it follows that the 0-set of coordinate axes can coincide with the f-set of coordinate axes by rotating the first around the Y_0-axis through an angle $(90° - \kappa_r)$.

A positive rotation is assumed and consequently the transformation matrix $T\kappa_r$ has the following form (see annex B) :

$$T\kappa_r = \begin{vmatrix} \sin\kappa_r & 0 & -\cos\kappa_r \\ 0 & 1 & 0 \\ \cos\kappa_r & 0 & \sin\kappa_r \end{vmatrix}$$

The transformed G_f-matrix is obtained by multiplying the matrices G and $T\kappa_r$:

$$G_f = G\,T\kappa_r = \begin{vmatrix} \cos\lambda_s \sin\kappa_r & \sin\lambda_s & -\cos\lambda_s \cos\kappa_r \\ \begin{bmatrix} -\sin\gamma_n \sin\lambda_s \sin\kappa_r \\ +\cos\gamma_n \cos\kappa_r \end{bmatrix} & \sin\gamma_n \cos\lambda_s & \begin{bmatrix} \sin\gamma_n \sin\lambda_s \cos\kappa_r \\ +\cos\gamma_n \sin\kappa_r \end{bmatrix} \end{vmatrix} \quad \ldots (1)$$

The matrix G_f contains the direction cosines of the cutting edge and of the intersection of the cutting edge normal plane and the face in the f-set of coordinate axes.

A.2.1.2 Transformation from the f-set of coordinate axes to the machine reference system (transformation L, H, G)

In the most general case, the two reference systems have an arbitrary orientation in relation to one another.

In 3.1, three elementary rotations are defined which when performed make the f-set of coordinate axes coincident with the machine reference system (see figure 3) :

- rotation around the Y-axis through the plan setting angle G;
- rotation around the new Z-axis through the elevation setting angle H;
- rotation around the new X-axis through the roll setting angle L.

The elementary transformation matrices, T_G, T_H and T_L are :

$$T_G = \begin{vmatrix} \cos G & 0 & \sin G \\ 0 & 1 & 0 \\ -\sin G & 0 & \cos G \end{vmatrix} \qquad T_H = \begin{vmatrix} \cos H & -\sin H & 0 \\ \sin H & \cos H & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T_L = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos L & -\sin L \\ 0 & \sin L & \cos L \end{vmatrix}$$

It must be noted that the tool angles are defined in relation to the f-set coordinate axes X_f, Y_f, Z_f and it is necessary to evaluate them in relation to the machine reference axes when the tool is positioned in the machine with the setting angles defined above. Consequently, the transformations must be performed in the inverse order using the inversed matrices. Since all the transformation matrices are orthogonal, the inversed matrices are obtained by interchanging rows and columns.

The complete transformation matrix which transforms the matrix G_f to the machine reference system is equal to the product of the elementary transformation matrices in the following sequence :

$$G_m = G_f\, T_L^t\, T_H^t\, T_G^t \quad \ldots (2)$$

A.2.1.3 Transformation from the machine reference system to the f_e-set of coordinate axes

1st way : using the motion angles M, N, T

The orientation of the f_e-set coordinate axes X_{fe}, Y_{fe}, Z_{fe} with respect to the machine references axes X_m, Y_m, Z_m is determined by the three motion angles M, N and T. The definitions of these angles are similar to those for the angles G, H and L (see 3.2).

The elementary transformation matrices T_M, T_N and T_T are :

$$T_M = \begin{vmatrix} \cos M & 0 & \sin M \\ 0 & 1 & 0 \\ -\sin M & 0 & \cos M \end{vmatrix} \qquad T_N = \begin{vmatrix} \cos N & -\sin N & 0 \\ \sin N & \cos N & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$T_T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos T & -\sin T \\ 0 & \sin T & \cos T \end{vmatrix}$$

The complete transformation matrix which transforms the matrix G_m into the f_e-set of coordinate axes is equal to the product of the elementary transformation matrices :

$$G_{fe} = G_m \, T_M \, T_N \, T_T \qquad \ldots (3)$$

2nd way : using the components of feed speed and cutting speed

Instead of using the motion angles, the position of the f_e-set of coordinate axes in relation to the machine setting system can be defined by the components of cutting speed and feed speed in the machine setting system. Indeed,

– the direction of the Y_{fe}-axis is according to its definition opposite to the resultant cutting direction. The latter is given by the components of the resultant cutting speed v_e, which have the following values :

$$(v_e)_{X_m} = (v_c)_{X_m} + (v_f)_{X_m}$$
$$(v_e)_{Y_m} = (v_c)_{Y_m} + (v_f)_{Y_m}$$
$$(v_e)_{Z_m} = (v_c)_{Z_m} + (v_f)_{Z_m}$$

By changing the signs of these components and dividing them by the absolute value of the resultant cutting speed, the direction cosines of the Y_{fe}-axis in the machine setting system are found.

– the X_{fe}-axis is perpendicular to the working plane P_{fe} defined by the direction of primary motion (cutting speed) and the direction of feed motion (feed speed).

The direction numbers of the P_{fe}-plane (and thus of a vector along the X_{fe}-axis) are :

$$(v_c)_{Y_m} (v_f)_{Z_m} - (v_c)_{Z_m} (v_f)_{Y_m}$$
$$(v_c)_{Z_m} (v_f)_{X_m} - (v_c)_{X_m} (v_f)_{Z_m}$$
$$(v_c)_{X_m} (v_f)_{Y_m} - (v_c)_{Y_m} (v_f)_{X_m}$$

The vector length is equal to the absolute value of the product

$$|\sin \varphi \, v_c \, v_f|$$

where

φ is the feed motion angle;

v_c is the cutting speed;

v_f is the feed speed.

By dividing the direction numbers given above by the vector length, the direction cosines of the X_{fe}-axis are found.

– the Z_{fe}-axis is perpendicular to the plane $X_{fe} - Y_{fe}$. For obtaining the direction cosines of the Z_{fe}-axis, the same procedure is followed as for the X_{fe}-axis. However, in this case the direction cosines of both the X_{fe}- and Y_{fe}-axes are known. Moreover, the two axes are perpendicular, so that the vector length along the Z_{fe}-axis is equal to 1.

The matrix containing the direction cosines of the X_{fe}-, Y_{fe}- and Z_{fe}-axes replaces the product $T_M \; T_N \; T_T$ in equation (3).

This transformation completes the left transformation array of figure 8 a).

A.2.2 Transformation from the 0_e-set of coordinate axes to the f_e-set of coordinate axes

The matrix G_e defined in the 0_e-set of coordinate axes is the starting point. The 0_e-set of coordinate axes can be made to coincide with the f_e-set of coordinate axes by rotation through an angle $(90^\circ - \kappa_{re})$ in the same manner as for the transformation from the 0-set of coordinate axes to the f-set of coordinate axes. Thus :

$$G'_{fe} = G_e \; T\kappa_{re} \qquad \ldots (4)$$

The resultant matrix G'_{fe} is identical in form to the matrix G_f (see expression 1) if the angles of the latter are provided with the additional suffix "e".

A.2.3 Equating procedure

Both the G_{fe} matrix and the G'_{fe} matrix contain the direction cosines of the cutting edge and the line of intersection of the cutting edge normal plane P_n and the face in relation to the f_e-set of coordinate axes. Both matrices must therefore be identical and their corresponding elements may be equated.

The resulting equations define the values of the working angles as a function of the tool angles, setting angles and motion angles resulting from the cutting conditions. Indeed, the elements of the G_{fe} matrix (right part of the equation) are dependent only on the tool angles λ_s, γ_n, κ_r, the setting angles G, H, L and the motion angles M, N, T. On the other hand, the elements of the G'_{fe} matrix (left part of the equation) are dependent only on the working angles λ_{se}, γ_{ne}, κ_{re}.

In order to simplify the expressions of the resultant formulae, a set of auxiliary angles is defined with the general notation (i, j). For this purpose the transformation between the f-set of coordinate axes and the f_e-set of coordinate axes is performed by only one transformation matrix T_{aux} :

$$T_{aux} = \begin{vmatrix} \cos(1,1) & \cos(1,2) & \cos(1,3) \\ \cos(2,1) & \cos(2,2) & \cos(2,3) \\ \cos(3,1) & \cos(3,2) & \cos(3,3) \end{vmatrix}$$

and

$$T_{aux} = T_L^t \; T_H^t \; T_G^t \; T_M \; T_N \; T_T \qquad \ldots (5)$$

From the last equation it follows that the values of cos (i, j) are as given in 4.3.

A.2.4 Note on the inverse transformation

Besides the direct transformation, where the working angles are determined as functions of the tool angles, an inverse transformation can be performed. The inverse transformation enables the tool angles to be determined if the working angles are given.

For this purpose, both the matrices G and G_e are transformed into the f-set of coordinate axes.

The transformation from the 0-set of coordinate axes into the f-set of coordinate axes is obtained by one rotation through the angle $(90^\circ - \kappa_r)$. The transformation from the 0_e-set of coordinate axes into the f-set of coordinate axes is performed by the following successive rotations : $(90^\circ - \kappa_{re})$, $-T$, $-N$, $-M$, G, H, L [see figure 8 b)].

It must be noted that between the f_e-set of coordinate axes and the f-set of coordinate axes, the transformation order and sign of the rotation angles are inversed with respect to the direct transformation, summarized in table 1.

The general formulae resulting from the inverse transformation are summarized in the second part of clause 4. They are obtained by equating the corresponding elements of the matrices G_{fe} and G'_{fe} :

$$G_{fe} = G'_{fe} \qquad \ldots (6)$$

where

$$G_{fe} = G\, T\kappa_r$$

$$G'_{fe} = G_e\, T\kappa_{re}\, T^t_T\, T^t_N\, T^t_M\, T_G\, T_H\, T_L$$

$$= G_e\, T\kappa_{re}\, T^t_{aux}$$

A.3 Derivation of the conversion formulae to give the incremental angles $(\gamma_{ne} - \gamma_n)$ and $(\alpha_n - \alpha_{ne})$

The line of intersection of the cutting edge normal plane P_n and the tool reference plane P_r is considered [see figure 8 c)] and the direction cosines of this line within the f-set of coordinate axes of axes X_f, Y_f, Z_f can be expressed in a matrix :

$$\begin{vmatrix} \cos\kappa_r & 0 & \sin\kappa_r \end{vmatrix}$$

The direction cosines of this same line within the f_e-set of coordinate axes X_{fe}, Y_{fe}, Z_{fe} can be obtained by multiplying the above matrix by the T_{aux} matrix :

$$\begin{vmatrix} \cos\kappa_r & 0 & \sin\kappa_r \end{vmatrix} \begin{vmatrix} \cos(1,1) & \cos(1,2) & \cos(1,3) \\ \cos(2,1) & \cos(2,2) & \cos(2,3) \\ \cos(3,1) & \cos(3,2) & \cos(3,3) \end{vmatrix}$$

$$= \begin{vmatrix} \begin{bmatrix} \cos\kappa_r \cos(1,1) \\ + \sin\kappa_r \cos(3,1) \end{bmatrix} & \begin{bmatrix} \cos\kappa_r \cos(1,2) \\ + \sin\kappa_r \cos(3,2) \end{bmatrix} & \begin{bmatrix} \cos\kappa_r \cos(1,3) \\ + \sin\kappa_r \cos(3,3) \end{bmatrix} \end{vmatrix}$$

If now a similar line of intersection of planes in the f_e-set of coordinate axes is considered, i.e. the intersection of the cutting edge normal plane P_{ne} and the working reference plane P_{re} [see figure 8 d)] the direction cosines of this line in relation to the f_e-set of coordinate axes of axes X_{fe}, Y_{fe}, Z_{fe} can be expressed in a matrix :

$$\begin{vmatrix} \cos\kappa_{re} & 0 & \sin\kappa_{re} \end{vmatrix}$$

The included angle between the two considered lines is the change in normal rake and is also the change in normal clearance. The cosine of $(\gamma_{ne} - \gamma_n)$ is given by taking the scalar product of the two considered lines such that :

$$\cos(\gamma_{ne} - \gamma_n) = \begin{vmatrix} \begin{bmatrix} \cos\kappa_r \cos(1,1) \\ + \sin\kappa_r \cos(3,1) \end{bmatrix} & \begin{bmatrix} \cos\kappa_r \cos(1,2) \\ + \sin\kappa_r \cos(3,2) \end{bmatrix} & \begin{bmatrix} \cos\kappa_r \cos(1,3) \\ + \sin\kappa_r \cos(3,3) \end{bmatrix} \end{vmatrix} \begin{vmatrix} \cos\kappa_{re} \\ 0 \\ \sin\kappa_{re} \end{vmatrix}$$

i.e. $\cos(\gamma_{ne} - \gamma_n) = \cos(\alpha_n - \alpha_{ne})$

$$= \cos\kappa_{re}\left[\cos\kappa_r \cos(1,1) + \sin\kappa_r \cos(3,1)\right] + \sin\kappa_{re}\left[\cos\kappa_r \cos(1,3) + \sin\kappa_r \cos(3,3)\right]$$

A.4 Derivation of the conversion formulae for working side and back rake and working side and back clearance

A vector perpendicular to the face is considered. Its magnitude is such that the component along the tool-in-hand Y_f-axis is – 1 [see figure 8 e)]. This vector in relation to the f-set of coordinate axes X_f, Y_f, Z_f may be expressed in a matrix :

$$\begin{vmatrix} \mathrm{tg}\,\gamma_p & -1 & \mathrm{tg}\,\gamma_f \end{vmatrix}$$

The relationship of the same vector with respect to the f_e-set of coordinate axes X_{fe}, Y_{fe}, Z_{fe} is obtained from

$$\begin{vmatrix} \operatorname{tg}\gamma_p & -1 & \operatorname{tg}\gamma_f \end{vmatrix} \begin{vmatrix} \cos(1,1) & \cos(1,2) & \cos(1,3) \\ \cos(2,1) & \cos(2,2) & \cos(2,3) \\ \cos(3,1) & \cos(3,2) & \cos(3,3) \end{vmatrix}$$

The elements of the resulting matrix are the components of the vector along the X_{fe}, Y_{fe} and Z_{fe} axes respectively [see figure 8 f)]. Thus it follows that :

$$\operatorname{tg}\gamma_{fe} = \frac{\operatorname{tg}\gamma_p \cos(1,3) - \cos(2,3) + \operatorname{tg}\gamma_f \cos(3,3)}{-\operatorname{tg}\gamma_p \cos(1,2) + \cos(2,2) - \operatorname{tg}\gamma_f \cos(3,2)}$$

and

$$\operatorname{tg}\gamma_{pe} = \frac{\operatorname{tg}\gamma_p \cos(1,1) - \cos(2,1) + \operatorname{tg}\gamma_f \cos(3,1)}{-\operatorname{tg}\gamma_p \cos(1,2) + \cos(2,2) - \operatorname{tg}\gamma_f \cos(3,2)}$$

From similar considerations it can be shown that :

$$\operatorname{ctg}\alpha_{fe} = \frac{\operatorname{ctg}\alpha_f \cos(3,3) + \operatorname{ctg}\alpha_p \cos(1,3) - \cos(2,3)}{-\operatorname{ctg}\alpha_f \cos(3,2) - \operatorname{ctg}\alpha_p \cos(1,2) + \cos(2,2)}$$

and

$$\operatorname{ctg}\alpha_{pe} = \frac{\operatorname{ctg}\alpha_f \cos(3,1) + \operatorname{ctg}\alpha_p \cos(1,1) - \cos(2,1)}{-\operatorname{ctg}\alpha_f \cos(3,2) - \operatorname{ctg}\alpha_p \cos(1,2) + \cos(2,2)}$$

Annex B

General method for obtaining the transformation matrices for elementary rotations around the X, Y and Z axes

It is assumed that the coordinates of a point in a system of coordinates X_1, Y_1, Z_1 are given. When it is required to determine the coordinates of the same point in a second system X_2, Y_2, Z_2 with the same origin as X_1, Y_1, Z_1, a transformation matrix is used. This transformation matrix is a quadratic orthogonal matrix of order 3.

For setting up a transformation matrix, the following rule applies : the rows of the matrix are respectively the direction cosines of the axes X_1, Y_1, Z_1 in the X_2, Y_2, Z_2 system of coordinates.

The expressions of the elements are very simple if both systems of coordinates possess a common axis.

Assume the axes X_1 and X_2 are coincident (see figure 9), then if x is the angle between the Y_1 and the Y_2 axis, the transformation matrix T_x can be written following the general rule given above :

$$T_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{vmatrix}$$

The coordinates of a point A, given in the X_1, Y_1, Z_1 system, are transformed to the X_2, Y_2, Z_2 system by multiplying the array of coordinates with the matrix T_x.

Example : If the coordinates of point A in X_1, Y_1, Z_1 are 0, 1, 1, the coordinates of point A in the X_2, Y_2, Z_2 system will be :

$$\begin{vmatrix} 0 & 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{vmatrix} = (0 \quad \cos x + \sin x \quad -\sin x + \cos x)$$

In the case where the Y-axes or the Z-axes are common, the respective transformation matrices T_y and T_z have the following expression :

$$T_y = \begin{vmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{vmatrix} \qquad T_z = \begin{vmatrix} \cos z & -\sin z & 0 \\ \sin z & \cos z & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

If further successive transformations have to be performed, the resultant transformation matrix is the product of the elementary transformation matrices.

Annex C

List of equivalent terms

No.	Symbol	English	French	Russian	German	Dutch
	—	Conversion formulae	Formules de conversion	Формулы превращения	Umrechnungsformein	Omrekeningsformules
	—	Direct transformation	Transformation directe	Прямое преобразование	direkte Transformation	Direkte transformatie
	—	Inverse transformation	Transformation inverse	Обратное преобразование	inverse Transformation	Inverse transformatie
2	—	Coordinate axes	Axes de coordonnées	Ось координат	Bezugssysteme	Referentiesystemen
2.1.1	$X_f - Y_f - Z_f$	f-set of coordinate axes	Système f d'axes de coordonnées	Система координатных осей-f	f-Bezugssystem	f-assenstelsel
2.1.2	$X_{fe} - Y_{fe} - Z_{fe}$	fe-set of coordinate axes	Système f_e d'axes de coordonnées	Система координатных осей-fe	fe-Bezuggsystem	fe-assenstelsel
2.2	$X_m - Y_m - Z_m$	Machine setting axes	Axes de position de la machine	Установочная ось станка	Maschinen-Einrichtsystem	Instelassen van de machine
3.1	—	Setting angles	Angles de position	Установочные углы	Einrichtwinkel	Instelhoeken
3.1.1	G	Plan setting angle	Angle de position en plan	Плоский установочный угол	Seiten-Einrichtwinkel	Vlakinstelhoek
3.1.2	H	Elevation setting angle	Angle de position en élévation	Вертикальный установочный угол	Höhen-Einrichtwinkel	Elevatieinstelhoek
3.1.3	L	Roll setting angle	Angle de position en pivotement	Установочный угол поворота	Dreh-Einrichtwinkel	Rolinstelhoek
3.2	—	Motion angles	Англес де моувемент	Углы движения	Wirk-Einrichtwinkel	Bewegingshoeken
3.2.1	M	Plan motion angle	Angle de mouvement en plan	Плоский угол движения	Wirk-Seiten-Einrichtwinkel	Vlakbewegingshoek
3.2.2	N	Elevation motion angle	Angle de mouvement en élévation	Вертикальный угол движения	Wirk-Höhen-Einrichtwinkel	Elevatiebewegingshoek
3.2.3	T	Roll motion angle	Англе де моувемент ен pivotement	Угол поворота движения	Wirk-Dreh-Einrichtwinkel	Rolvewegingshoek
4.3	(i, j)	Auxiliary angles	Angles auxiliaires	Вспомогательные углы	Hilfswinkel	Hulphoeken

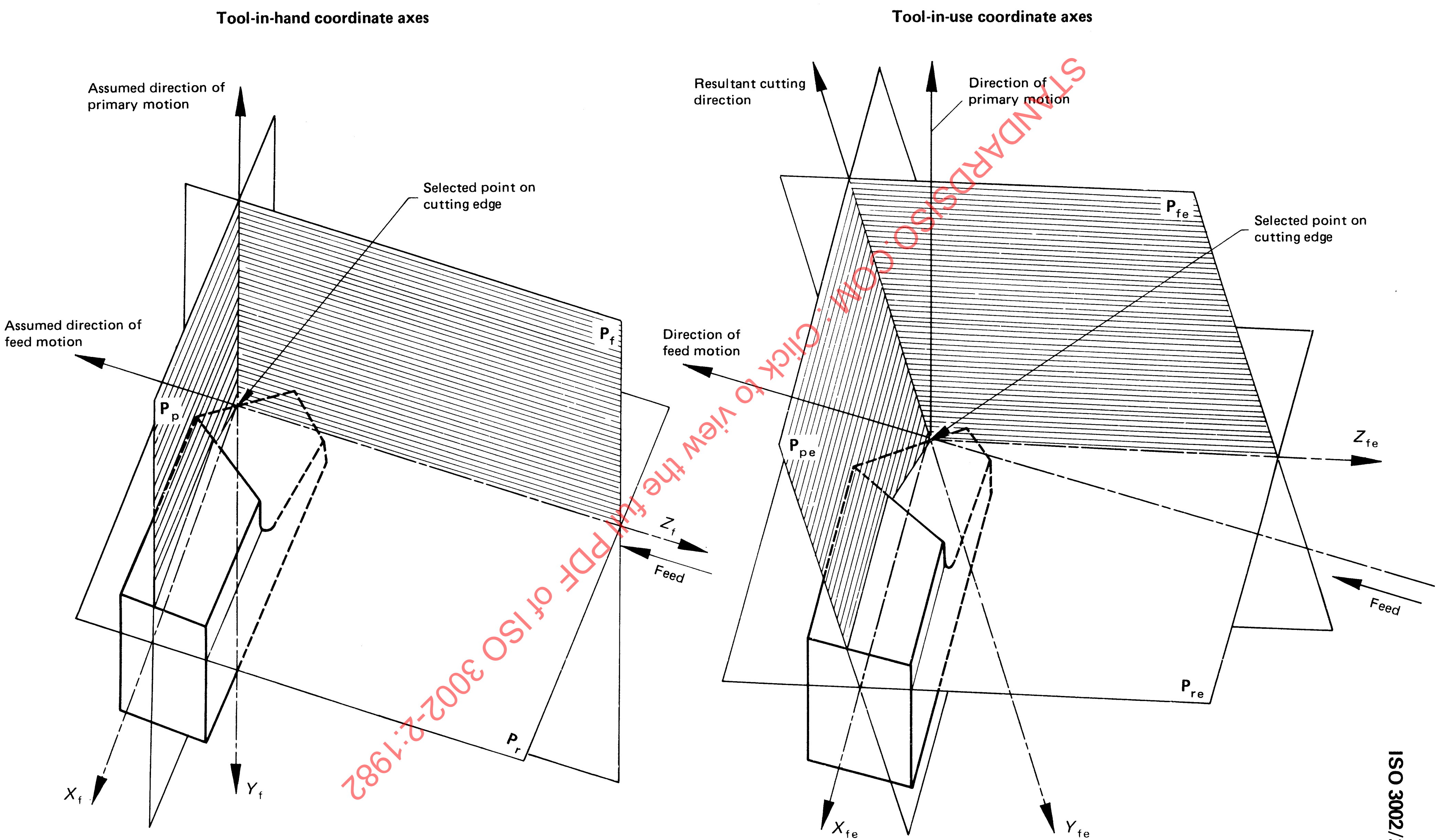

Figure 1 — Right-hand turning tool

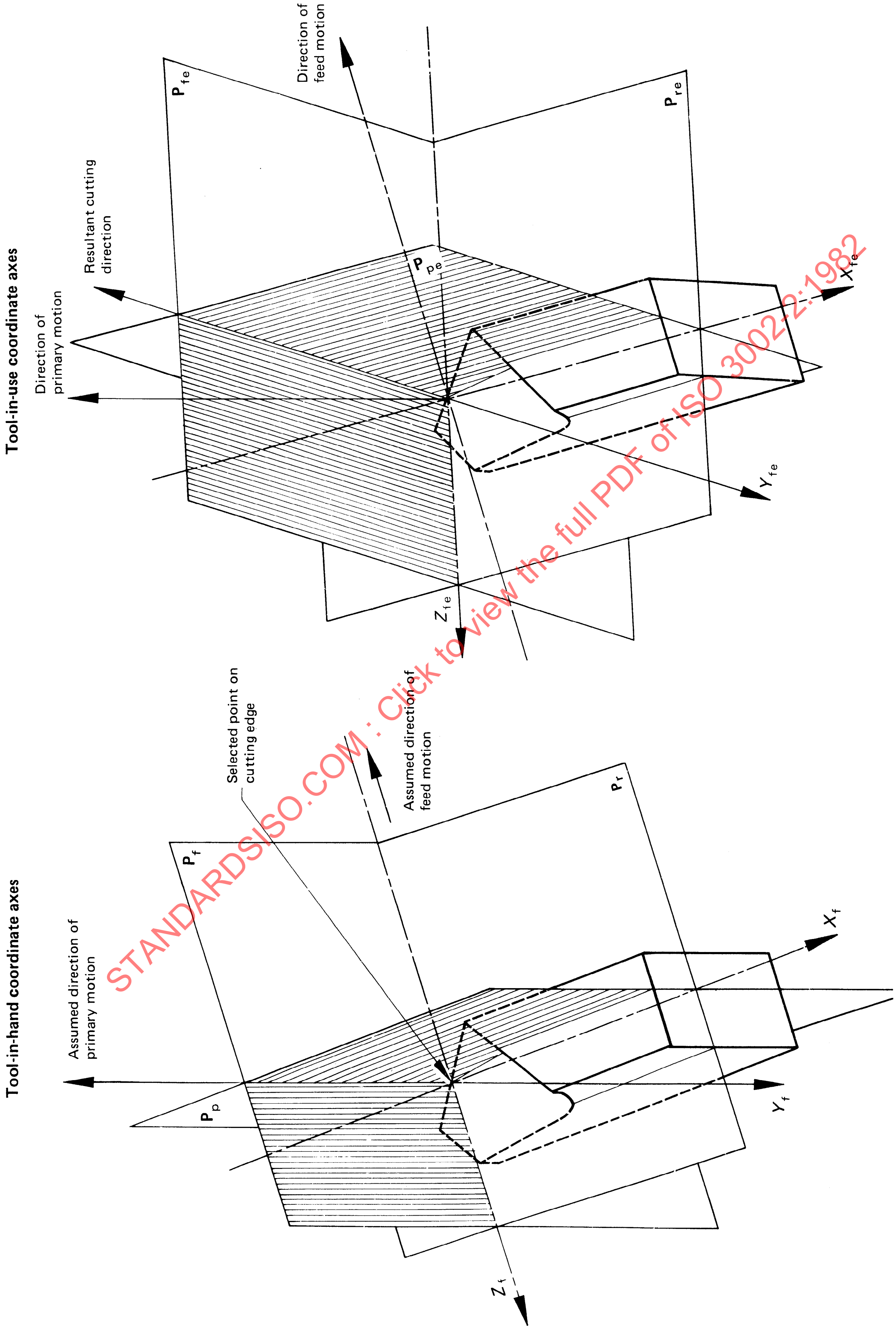

Figure 2 — Left-hand turning tool

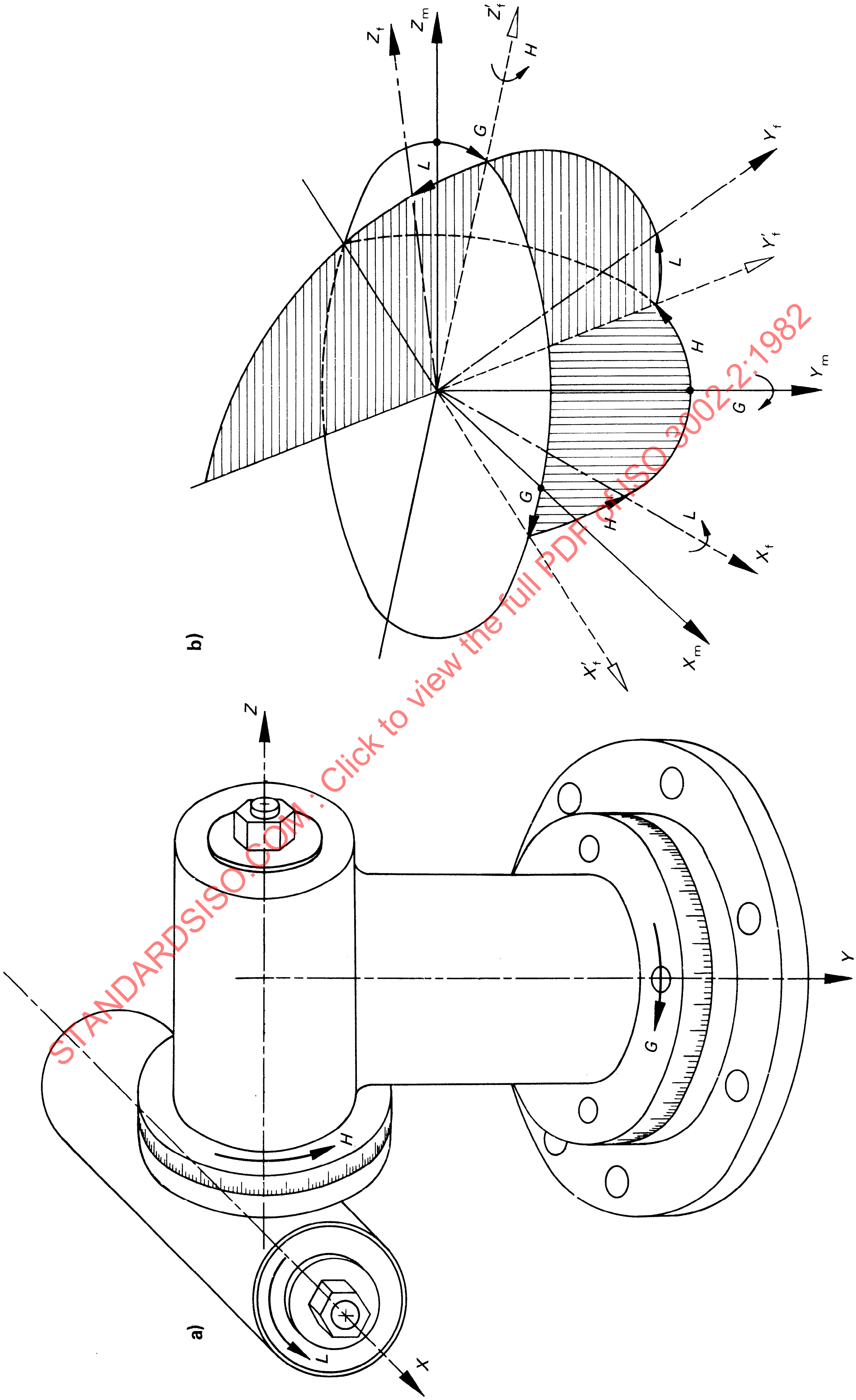

Figure 3 — Setting angles

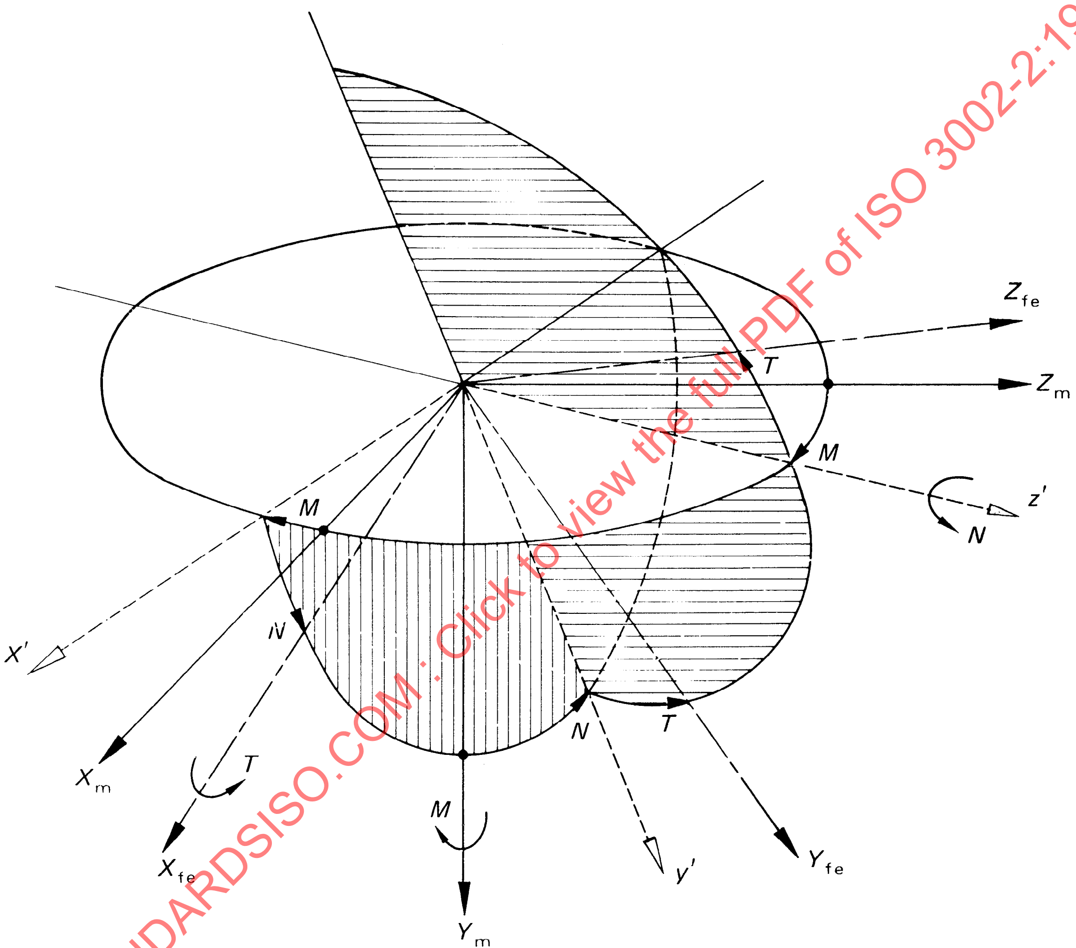

Figure 4 — Motion angles