INTERNATIONAL STANDARD

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Optics and optical instruments — Geodetic instruments — Field procedures for determining accuracy —

Part 2:
Theodolites

Optique et instruments d'optique — Instruments géodésiques — Méthodes de détermination sur site de la précision —

Partie 2: Théodolites

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 12857-2 was prepared by Technical Committee ISO/TC 172, *Optics and optical instruments*, Subcommittee SC 6, *Geodetic and surveying instruments.*

ISO 12857 consists of the following parts, under the general title *Optics and optical instruments — Geodetic instruments — Field procedures for determining accuracy*:

— *Part 1: Levels*

— *Part 2: Theodolites*

— *Part 3: Electro-optical distance meters (EDM instruments)*

Annexes A and B of this part of ISO 12857 are for information only.

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Optics and optical instruments — Geodetic instruments — Field procedures for determining accuracy —

Part 2:
Theodolites

1 Scope

This part of ISO 12857 specifies field procedures to be adopted when determining and assessing the accuracy of theodolites used in surveying.

These tests are intended to be operational and not tests for acceptance or performance.

The procedures are applicable to the determination of the accuracy of different instruments at one time or of one instrument at different times.

The field procedures can be applied everywhere without the need of special ancillary equipment and are designed to minimize atmospheric influences.

NOTE — Other International Standards for testing measuring instruments for building construction are available.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO 12857. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO 12857 are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 9849:1991, *Optics and optical instruments — Geodetic instruments — Vocabulary*

3 Definitions

For the purposes of this part of ISO 12857, the terms and definitions given in ISO 3534-1 and ISO 9849 apply.

4 General

The theodolite and its ancillary equipment shall be in known and acceptable states of adjustment by the user according to methods specified in the manufacturers' handbooks.

The accuracy of theodolites is expressed in terms of the standard deviation of a horizontal direction (HZ), observed once in both face positions of the telescope, or of a vertical angle (V) observed once in both face positions of the telescope.

The test procedures given in this part of ISO 12857 are intended for determining the standard deviations $s_{\text{ISO-THEO-HZ}}$ and $s_{\text{ISO-THEO-V}}$.

Statistical tests should be applied to determine whether the standard deviation s obtained belongs to the population of the instrumentation's standard deviation, whether two tested samples belong to the same population, or whether the index correction o of the vertical circle is zero.

5 Procedures

5.1 Measurement of horizontal directions

5.1.1 General

The following field procedures shall be adopted for determining the accuracy of theodolites for horizontal directions, by a single survey team with a single instrument and its ancillary equipment.

The results of these tests are influenced by meteorological conditions. These conditions will include different air temperatures and pressures, wind speed, cloud cover and visibility. An overcast sky guarantees the most favourable weather conditions. Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high and therefore they are not practicable for most users.

5.1.2 Test configuration

Five targets shall be set up located in approximately the same horizontal plane as the instrument, between 100 m and 250 m away, and situated at intervals around the horizon as regular as possible. Targets shall be used that can be observed unmistakably, preferably target plates.

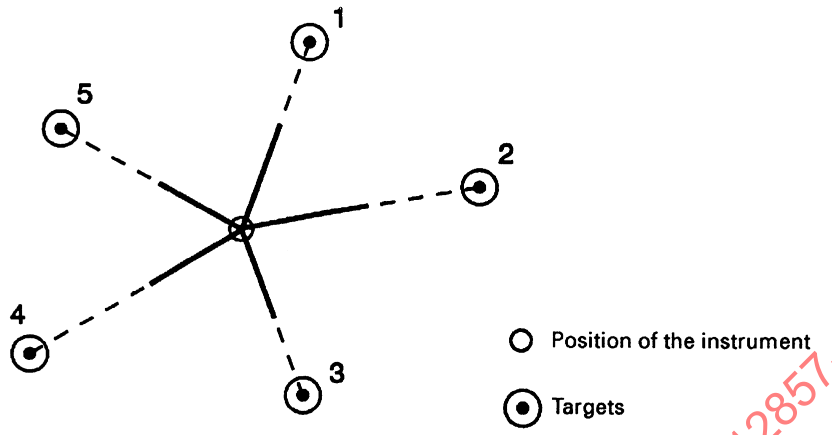

Figure 1 — Test configuration

5.1.3 Observations

Four series of observations shall be performed. The observations shall be carried out under various but not extreme weather conditions. Each series (k) of observations shall consist of three sets (j) of directions to the five targets (i).

The five targets shall be observed in each set in face position I of the telescope in a clockwise sequence (1), (2), (3), (4), (5), and in face position II of the telescope in an anticlockwise sequence (5), (4), (3), (2), (1). The graduated circle shall be changed for 60° (67 gon) after each set. If physical rotation of the graduated circle is not possible, e.g. for electronic theodolites, the lower part of the theodolite may be turned approximately 120° (133 gon) on the tribrach.

5.1.4 Calculations

The evaluation of the measured values is an adjustment of observation equations. Within a series of observations k, one direction is marked by $r_{i,j,\mathrm{I}}$ or $r_{i,j,\mathrm{II}}$, the index i being the target and the index j being the number of the set. I or II points out the face position of the telescope. Each series of observations is evaluated separately.

First of all the mean values

$$r_{i,j} = \frac{r_{i,j,\mathrm{I}} + r_{i,j,\mathrm{II}} \pm 200\,\mathrm{gon}}{2} \; ; \; i = 1,2,3,4,5 \; ; \; j = 1,2,3$$

of the readings in both face positions I and II of the telescope are calculated. By reduction into the direction to target No. 1 we obtain the results:

$$r'_{i,j} = r_{i,j} - r_{1,j}; \quad i = 1,2,3,4,5; \quad j = 1,2,3$$

The mean values of the directions resulting from three sets to target No. i are:

$$\bar{r}_i = \frac{r'_{i,1} + r'_{i,2} + r'_{i,3}}{3} \quad ; \quad i = 1,2,3,4,5$$

From the differences

$$d_{i,j} = \bar{r}_i - r'_{i,j} \quad ; \quad i = 1,2,3,4,5 \; ; \; j = 1,2,3$$

we obtain for each set of observations the arithmetic mean values

$$\bar{d}_j = \frac{d_{1,j} + d_{2,j} + d_{3,j} + d_{4,j} + d_{5,j}}{5} \quad ; \quad j = 1,2,3$$

from which the corrections result:

$$c_{i,j} = d_{i,j} - \bar{d}_j ; \quad i = 1,2,3,4,5; \quad j = 1,2,3$$

Except for the rounding errors, each set must meet the condition:

$$\sum_{i=1}^{5} c_{i,j} = 0 \; ; \quad j = 1,2,3$$

The square sum of the corrections of the series of observations No. k is:

$$cc_k = \sum_{j=1}^{3} \sum_{i=1}^{5} c_{i,j}^2$$

For three sets of directions to five targets in each case the degree of freedom is:

$$f_k = (3-1) \cdot (5-1) = 8$$

and the standard deviation s_k of a direction $r_{i,j}$, taken in one set of the series of observations No. k, amounts to:

$$s_k = \sqrt{\frac{cc_k}{f_k}} = \sqrt{\frac{cc_k}{8}}$$

The standard deviation s_0 of a horizontal direction observed in one set (arithmetic mean of the readings in both face positions of the telescope) according to this part of ISO 12857, calculated from all four series of observations at a degree of freedom of

$$f = 4; \quad f_k = 32$$

amounts to:

$$s_0 = \sqrt{\frac{\sum_{k=1}^{4} cc_k}{f}} = \sqrt{\frac{\sum_{k=1}^{4} cc_k}{32}}$$

$$s_{\text{ISO-THEO-HZ}} = s_0$$

5.1.5 Statistical tests

For interpretation of the results, statistical tests shall be carried out using the standard deviation s_0 of a horizontal direction observed in one set in order to answer the following questions.

A) Is the calculated standard deviation $s_{ISO\text{-}THEO\text{-}HZ}$ smaller than the value σ_0 stated by the manufacturer or smaller than another predetermined value σ_0 ?

B) Do two standard deviations s_1 and s_2, as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same degree of freedom f?

The standard deviations s_1 and s_2 may be obtained from:

- two series of measurements by the same instrument but different observers;
- two series of measurements by the same instrument at different times;
- two series of measurements by different instruments.

Table 1 - Statistical tests

Question	Null hypothesis	Alternative hypothesis
A	$s_0 \leq \sigma_0$	$s_0 > \sigma_0$
B	$\sigma_1 = \sigma_2$	$\sigma_1 \neq \sigma_2$

For the following tests a confidence level of $1-\alpha = 0{,}95$ and according to the design of measurements a degree of freedom f = 32 are assumed.

A) The null hypothesis stating that the empirically determined standard deviation s_0 is smaller than or equal to a theoretical or predetermined value σ_0 is not rejected if the following condition is fulfilled:

$$s_0 \leq \sigma_0 \cdot \sqrt{\frac{\chi^2_{f,1-\alpha}}{f}}$$

$$s_0 \leq \sigma_0 \cdot \sqrt{\frac{\chi^2_{32;0,95}}{32}}$$

$$\chi^2_{32;0,95} = 46{,}19$$

$$s_0 \leq \sigma_0 \cdot \sqrt{\frac{46{,}19}{32}}$$

$$s_0 \leq \sigma_0 \cdot 1{,}20$$

Otherwise, the null hypothesis is rejected.

B) In the case of two different samples No. 1 and No. 2, a test indicates whether the estimated standard deviations s_1 and s_2 belong to the same population. The corresponding null hypothesis $\sigma_1 = \sigma_2$ is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{f,f,1-\alpha/2}} \leq \frac{s_1^2}{s_2^2} \leq F_{f,f,1-\alpha/2}$$

$$\frac{1}{F_{32;32;0,975}} \leq \frac{s_1^2}{s_2^2} < F_{32;32;0,975}$$

$$F_{32;32;0,975} = 2,02$$

$$0,49 \leq \frac{s_1^2}{s_2^2} \leq 2,02$$

Otherwise, the null hypothesis is rejected.

The degree of freedom and, thus, the corresponding test values $\chi^2_{f,\,1-\alpha}$ and $F_{f_1,f_2,\,1-\alpha 2}$ (taken from reference books on statistics) change if a different number of observations is analysed.

5.2 Measurements of vertical angles

5.2.1 General

The following field procedures shall be adopted for determining the accuracy of theodolites for vertical angles measured by a single survey team with a single instrument and its ancillary equipment.

The results of these tests are influenced by meteorological conditions. These conditions will include different air temperatures and pressures, wind speed, cloud cover and visibility. An overcast sky guarantees the most favourable weather conditions. Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high and therefore they are not practicable for most users.

5.2.2 Test configuration

A precision invar levelling staff shall be set up vertically in a fixed position by means of struts during every series of observations. When testing a theodolite with a resolution of 3" (1 mgon) the distance x_2 should be approx. 5 m, and for theodolites with a resolution of 1" (0,3 mgon) approx. 15 m (see figure 2).

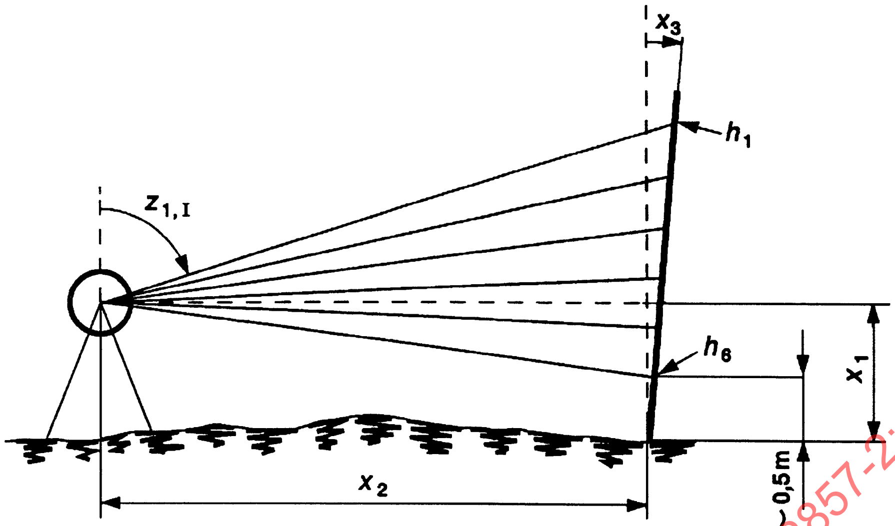

Figure 2 - Test configuration

5.2.3 Observations

In total four series of observations shall be carried out under various but not extreme weather conditions. The height difference between the instrument and the staff shall be varied in order to cover a larger range of the vertical circle.

Each series of observations (k) shall comprise twelve vertical angle (zenith angle) measurements taken to six graduation lines (i) on the precision invar staff, six angles in face position I and six angles in face position II of the telescope. The graduation lines shall be spread evenly over the staff, but sights near the ground shall be avoided.

5.2.4 Calculations

The evaluation of the measured angles is an adjustment of observation equations. Within a series of observations No. k, one zenith angle is denoted by $z_{i,\mathrm{I}}$ or $z_{i,\mathrm{II}}$, the index i being the graduation line. I or II points out the face position of the telescope. Each series of observations is evaluated separately.

NOTE – For facilitating the following calculations, a suitable computer program is given in Annex B of this part of ISO 12857.

In the calculations all distances are in centimetres. The units of the observations z depend on the type of the theodolite. If the reading is centesimal grades (gon) then we take for $\rho = 200$ gon/π . If the reading is sexagesimal degrees (°), then we take $\rho = 180°/\pi$.

The non-linear observation equations of the adjustment are for measurement in face position I

$$z_{i,\mathrm{I}} + c_{i,\mathrm{I}} + o = \rho \cdot \left(\frac{\pi}{2} - \arctan\left(\frac{h_i \cdot \cos x_3 - x_1}{x_2 + h_i \cdot \sin x_3} \right) \right); \qquad i = 1, \ldots ,6$$

and for measurements in face position II

$$z_{i,\mathrm{I}} + c_{i,\mathrm{I}} + o = \rho \cdot \left(\frac{3\pi}{2} + \arctan\left(\frac{h_i \cdot \cos x_3 - x_1}{x_2 + h_i \cdot \sin x_3} \right) \right); \qquad i = 1, \ldots ,6$$

where

- o is the unknown index correction (orientation of the vertical circle);
- x_1 is the height of the tilting axis of the theodolite, related to the zero-point of the levelling staff;
- x_2 is the distance of the theodolite from the levelling staff;
- x_3 is the inclination of the levelling staff against the vertical direction (angle between the zenith and the levelling staff, this angle shall be the same over one series of observation).

Since the functional model is not linear in the three unknown parameters x_1 , x_2 and x_3, it is necessary to determine approximate values x_1^0 and x_2^0. The approximate value for x_3 is zero. x_1^0 and x_2^0 should be measured directly. Alternatively, they can be determined by measurements in face I to the upper line h_1 and to the lowest line h_6 of the graduation:

$$x_1^0 = h_6 - \cos z_{6,\mathrm{I}} \cdot \frac{(h_1 - h_6) \cdot \sin z_{1,\mathrm{I}}}{\sin(z_{6,\mathrm{I}} - z_{1,\mathrm{I}})}$$

$$x_2^0 = \sin z_{6,\mathrm{I}} \cdot \frac{(h_1 - h_6) \cdot \sin z_{1,\mathrm{I}}}{\sin(z_{6,\mathrm{I}} - z_{1,\mathrm{I}})}$$

In order to get normal equations with appropriate units, we use in the following linearized observation equations for the two angles to graduation line No. i a scaling factor fa, which is for centesimal grades $fa = 100$ and for sexagesimal degrees $fa = 60$:

$$c_{i,\mathrm{I}} = -o + a_{i,1} dx_1 + a_{i,2} dx_2 - a_{i,3} x_3 - l_{i,\mathrm{I}}$$

$$c_{i,\mathrm{II}} = -o - a_{i,1} dx_1 - a_{i,2} dx_2 - a_{i,3} x_3 - l_{i,\mathrm{II}}$$

with

$$a_{i,1} = \mathrm{r} \cdot \frac{x_2^0}{(h_i - x_1^0)^2 + (x_2^0)^2} \cdot fa$$

$$a_{i,2} = \mathrm{r} \cdot \frac{h_i - x_1^0}{(h_i - x_1^0)^2 + (x_2^0)^2} \cdot fa$$

$$a_{i,3} = \frac{h_i \cdot a_{i,2}}{\mathrm{r}}$$

$$l_{i,\mathrm{I}} = \left(z_{i,\mathrm{I}} - \mathrm{r} \cdot \left(\frac{\mathrm{p}}{2} - \arctan \frac{h_i - x_1^0}{x_2^0} \right) \right) \cdot fa$$

$$l_{i,\mathrm{II}} = \left(z_{i,\mathrm{II}} - \mathrm{r} \cdot \left(\frac{3\mathrm{p}}{2} + \arctan \frac{h_i - x_1^0}{x_2^0} \right) \right) \cdot fa$$

$$x_1 = x_1^0 + dx_1$$
$$x_2 = x_2^0 + dx_2$$

With the twelve observation equations the normal equations are formed. If the observations in both face positions of the telescope were taken symmetrically the system of the normals consists of two independent parts:

$$12\,o = -\sum_{i=1}^{6}(l_{i,\mathrm{I}} + l_{i,\mathrm{II}})$$
$$2\mathbf{A}^{\mathrm{T}}\mathbf{A}\cdot\mathbf{x} = \mathbf{A}^{\mathrm{T}}(\mathbf{l}_{\mathrm{I}} - \mathbf{l}_{\mathrm{II}})$$

with

$$\mathbf{A} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ \cdots & \cdots & \cdots \\ a_{6,1} & a_{6,2} & a_{6,3} \end{pmatrix} \quad \mathbf{l}_{\mathrm{I}} = \begin{pmatrix} l_{1,\mathrm{I}} \\ \cdots \\ l_{6,\mathrm{I}} \end{pmatrix} \quad \mathbf{l}_{\mathrm{II}} = \begin{pmatrix} l_{1,\mathrm{II}} \\ \cdots \\ l_{6,\mathrm{II}} \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} dx_1 \\ dx_2 \\ x_3 \end{pmatrix}$$

They determine the four parameters:

$$o = -\frac{1}{12}\sum_{i=1}^{6}(l_{i,\mathrm{I}} + l_{i,\mathrm{II}})$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1^0 \\ x_2^0 \\ 0 \end{pmatrix} + \begin{pmatrix} dx_1 \\ dx_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1^0 \\ x_2^0 \\ 0 \end{pmatrix} + (2\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\cdot\mathbf{A}^{\mathrm{T}}(\mathbf{l}_{\mathrm{I}} - \mathbf{l}_{\mathrm{II}})$$

The scaling with *fa* gives *o* in cgon or in (ϑ, dx_1 and dx_2 in cm, x_3 in grads (gon) or degrees (°).

The square sum cc_k of the corrections of series No. *k* for the determination of the standard deviation s_k is calculated in two different ways, thus giving an effective check.

$$cc_k = (\mathbf{l}_{\mathrm{I}}^{\mathrm{T}}\cdot\mathbf{l}_{\mathrm{I}} + \mathbf{l}_{\mathrm{II}}^{\mathrm{T}}\cdot\mathbf{l}_{\mathrm{II}}) - 12o^2 - (\mathbf{l}_{\mathrm{I}}^{\mathrm{T}} - \mathbf{l}_{\mathrm{II}}^{\mathrm{T}})\mathbf{A}\mathbf{x}$$

All terms of this equation are in cgon or (ϑ.

Alternatively, starting from the non-linear basic equations the corrections $c_{i,\mathrm{I}}$ and $c_{i,\mathrm{II}}$ are calculated for $i = 1,\ldots,6$, squared and added:

$$c_{i,\mathrm{I}} = \mathrm{r}\cdot\left(\frac{\mathrm{p}}{2} - arctan\frac{h_i\cdot cos\,x_3 - x_1}{x_2 + h_i\cdot sin\,x_3}\right) - o - z_{i,\mathrm{I}}$$

$$c_{i,\mathrm{II}} = \mathrm{r}\cdot\left(\frac{3\mathrm{p}}{2} + arctan\frac{h_i\cdot cos\,x_3 - x_1}{x_2 + h_i\cdot sin\,x_3}\right) - o - z_{i,\mathrm{II}}$$

$$cc_k = \sum_{i=1}^{6}\left(c_{i,\mathrm{I}}^2 + c_{i,\mathrm{II}}^2\right)$$

Except for rounding errors, the same result should be obtained.

For each series of observations there are twelve measurements for the determination of four unknown parameters. Thus the degree of freedom is

$$f_k = 12 - 4 = 8$$

The standard deviation of a zenith angle of the series of observations No. k once observed in one face position amounts to

$$s_k = \sqrt{\frac{cc_k}{f_k}} = \sqrt{\frac{cc_k}{8}}$$

For the standard deviation s_0 calculated from all series of observations the degree of freedom is

$$f = 4 f_k = 32$$

and the standard deviation of unit weight

$$s_0 = \sqrt{\frac{\sum_{k=1}^{4} cc_k}{f}} = \sqrt{\frac{\sum_{k=1}^{4} cc_k}{32}} = \sqrt{\frac{\sum_{k=1}^{4} s_k^2}{4}}$$

The standard deviation of a vertical angle observed once, but in both face positions, calculated from all series of observations is

$$s_{\text{ISO-THEO-V}} = \frac{s_0}{\sqrt{2}}$$

5.2.5 Statistical tests

For interpretation of results, statistical tests shall be carried out using

- the standard deviation s_0 of a vertical angle observed in one face position
- the orientation o of the vertical circle

in order to answer the following questions.

A) Is the computed standard deviation $s_{\text{ISO-THEO-V}}$ smaller than the value σ stated by the manufacturer or smaller than another predetermined value σ ?

B) Do two standard deviations s_1 and s_2, as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same degree of freedom f?

The standard deviations s_1 and s_2 may be obtained from:
- two series of measurements by the same instrument but different observers;
- two series of measurements by the same instrument at different times;
- two series of measurements by different instruments.

C) Is the index correction o equal to zero?

Table 2 - Statistical tests

Question	Null hypothesis	Alternative hypothesis
A	$s_0 / \sqrt{2} \le \sigma$	$s_0 / \sqrt{2} > \sigma$
B	$\sigma_1 = \sigma_2$	$\sigma_1 \ne \sigma_2$
C	$o = 0$	$o \ne 0$

For the following tests a confidence level of $1-a=0{,}95$ and according to the design of measurements a degree of freedom of $f = 32$ is assumed.

A) The null hypothesis stating that the empirically determined standard deviation $s_{\text{ISO-THEO-V}}$ of a vertical angle observed in both face positions is smaller than or equal to a theoretical or a predetermined value σ is not rejected if the following condition is fulfilled:

$$s_0 \le s_0 \cdot \sqrt{\frac{C^2_{f,\,1-a}}{f}}$$

$$s_0 \le s_0 \cdot \sqrt{\frac{C^2_{32;\ 0{,}95}}{32}}$$

$$s_0 \le s_0 \cdot \sqrt{\frac{46{,}19}{32}}$$

$$s_0 \le s_0 \cdot 1{,}20$$

Otherwise, the null hypothesis is rejected.

B) In the case of two different samples No. 1 and No. 2 a test indicates whether the estimated standard deviations s_1 and s_2 belong to the same population. The corresponding null hypothesis $s_1 = s_2$ is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{f_1, f_2, 1-\frac{a}{2}}} < \frac{s_1^2}{s_2^2} < F_{f_1, f_2, 1-\frac{a}{2}}$$

$$\frac{1}{F_{32;\,32;\,0{,}975}} < \frac{s_1^2}{s_2^2} < F_{32;\,32;\,0{,}975}$$

$$0{,}49 < \frac{s_1^2}{s_2^2} < 2{,}02$$

Otherwise, the null hypothesis is rejected.

C) The hypothesis stating that the index correction o is equal to zero is not rejected if the following condition is fulfilled:

$$|o| \leq s_o \cdot t_{f,1-\frac{a}{2}}$$

$$|o| \leq s_o \cdot t_{32;\,0,975}$$

$$t_{32;\,0,975} = 2{,}04$$

$$s_o = \frac{s_o}{\sqrt{12}}$$

$$|o| \leq \frac{s_o}{\sqrt{12}} \cdot 2{,}04$$

Otherwise, the null hypothesis is rejected.

The degree of freedom and, thus, the corresponding test values $\chi^2_{f,1-\alpha}$, $F_{f,f,1-\alpha/2}$ and $t_{f,1-\alpha/2}$ (taken from reference books on statistics) change if a different number of observations is analysed.

Annex A
(informative)

Examples of calculations

A.1 Horizontal directions

A.1.1 Observations

For the series of observations No. 1 the horizontal directions with two different theodolite types (designated as A and B) are given in tables A.1 and A.2.

Table A.1 - Example of field observations and evaluation for theodolite type A

j	i	$r_{i,j,\mathrm{I}}$ ° ' "	$r_{i,j,\mathrm{II}}$ ° ' "	$r_{i,j}$ ° ' "	$r'_{i,j}$ ° ' "	$\bar{r}_i$ ° ' "	$d_{i,j}$ "	$c_{i,j}$ "	$c^2_{i,j}$ (")²
1	1	28 12 37	208 12 42	28 12 39,5	0 00 00,0	0 00 00,0	0,0	0,1	0,01
	2	83 50 35	263 50 40	83 50 37,5	55 37 58,0	55 38 00,3	2,3	2,4	5,76
	3	141 45 30	321 45 35	141 45 32,5	113 32 53,0	113 32 50,8	-2,2	-2,1	4,41
	4	219 30 49	39 30 50	219 30 49,5	191 18 10,0	191 18 9,5	-0,5	-0,4	0,16
	5	308 26 31	128 26 33	308 26 32,0	280 13 52,5	280 13 52,5	0,0	0,1	0,01
	Σ						-0,4	0,1	
2	1	87 48 51	267 48 55	87 48 53,0	0 00 00,0		0,0	-1,7	2,89
	2	143 26 52	323 26 51	143 26 51,5	55 37 58,5		1,8	0,1	0,01
	3	201 21 41	21 21 47	201 21 44,0	113 32 51,0		-0,2	-1,9	3,61
	4	279 07 01	99 06 59	279 07 00,0	191 18 07,0		2,5	0,8	0,64
	5	8 02 42	188 02 40	8 02 41,0	280 13 48,0		4,5	2,8	7,84
	Σ						8,6	0,1	
3	1	147 08 13	327 08 08	147 08 10,5	0 00 00,0		0,0	1,7	2,89
	2	202 46 17	22 46 13	202 46 15,0	55 38 04,5		-4,2	-2,5	6,25
	3	260 41 01	80 40 57	260 40 59,0	113 32 48,5		2,3	4,0	16,00
	4	338 26 24	158 26 20	338 26 22,0	191 18 11,5		-2,0	-0,3	0,09
	5	67 22 07	247 22 08	67 22 07,5	280 13 57,0		-4,5	-2,8	7,84
	Σ						-8,4	0,1	58,41

NOTE - The series of observations No. 2, 3 and 4 are not printed.

Table A.2 - Example of field observations and evaluation for theodolite type B

j	i	$r_{i,j,\text{I}}$ gon	$r_{i,j,\text{II}}$ gon	$r_{i,j}$ gon	$r'_{i,j}$ gon	$\bar{r}_i$ gon	$d_{i,j}$ mgon	$c_{i,j}$ mgon	$c^2_{i,j}$ mgon²
1	1	310,475	110,470	310,472	0,000	0,000	0	–2	4
	2	366,131	166,126	366,128	55,656	55,655	–1	–3	9
	3	30,481	230,477	30,479	120,007	120,006	–1	–3	9
	4	98,878	298,872	98,875	188,403	188,405	2	0	0
	5	210,347	10,341	210,344	299,872	299,880	8	6	36
	Σ						8	–2	
2	1	376,749	176,744	376,746	0,000		0	1	1
	2	32,403	232,398	32,400	55,654		1	2	4
	3	96,753	296,749	96,751	120,005		1	2	4
	4	165,154	365,148	165,151	188,405		0	1	1
	5	276,638	76,630	276,634	299,888		–8	–7	49
	Σ						–6	–1	
3	1	42,049	242,044	42,046	0,000		0	0	0
	2	97,705	297,700	97,702	55,656		–1	–1	1
	3	162,056	362,050	162,053	120,007		–1	–1	1
	4	230,454	30,449	230,452	188,406		–1	–1	1
	5	341,929	141,921	341,925	299,879		1	1	1
	Σ						–2	–2	121

NOTE - The series of observations No. 2, 3 and 4 are not printed.

A.1.2 Results

Results for series of observations No. 1 for theodolite type A (see table A.1):

$$cc_1 = 58{,}41\ ('')^2$$

$$s_1 = \sqrt{\frac{58{,}41}{8}} \cdot 1'' = 2{,}7''$$

1st series of observations: $s_1 = 2{,}7''$
2nd series of observations: $s_2 = 2{,}0''$
3rd series of observations: $s_3 = 1{,}6''$
4th series of observations: $s_4 = 2{,}3''$

$$s_0 = \sqrt{\frac{19{,}14}{4}} \cdot 1'' = 2{,}2''$$

$$s_{\text{ISO-THEO-HZ}} = 2{,}2''$$

Results for series of observations No. 1 for theodolite B (see table A.2):

$cc_1 = 121 \text{ mgon}^2$

$$s_1 = \sqrt{\frac{121}{8}} \text{ mgon} = 3{,}9 \text{ mgon}$$

1st	series of observations:	$s_1 = 3{,}9$ mgon
2nd	series of observations:	$s_2 = 2{,}7$ mgon
3rd	series of observations:	$s_3 = 3{,}1$ mgon
4th	series of observations:	$s_4 = 1{,}7$ mgon

$$s_0 = \sqrt{\frac{35{,}0}{4}} \text{ mgon} = 3{,}0 \text{ mgon}$$

$$s_{\text{ISO-THEO-HZ}} = 3{,}0 \text{ mgon}$$

A.1.3 Statistical tests

Example for a test according to question A for theodolite type A:

$$\sigma_0 = 2{,}0''$$
$$2{,}2 \leq 2{,}0'' \cdot 1{,}20$$

Since the above inequality is true, the null hypothesis stating that the empirically determined standard deviation $s_0 = 2{,}2''$ is smaller than or equal to the manufacturer's value $\sigma_0 = 2{,}0''$ is not rejected.

Example for a test according to question B for theodolite type B:

$$s_1 = 3{,}0 \text{ mgon}$$
$$s_2 = 1{,}9 \text{ mgon}$$
$$0{,}49 \leq \frac{9{,}00 \text{ mgon}^2}{3{,}61 \text{ mgon}^2} \leq 2{,}02$$
$$0{,}49 \leq 2{,}49 \leq 2{,}02$$

Since the above inequality is not true, the null hypothesis stating that the standard deviations $s_1 = 3{,}0$ mgon and $s_2 = 1{,}9$ mgon obtained in the same manner from measurements by the same instrument at different times, belong to the same population is rejected.

A.2 Vertical directions

A.2.1 Observations

For the series of observations No. 1 the vertical angles with two different theodolite types (designated as A and B) are given in tables A.3 and A.4.

Table A.3 - Example of field observations for theodolite type A

i	h_i cm	$z_{i,I}$ gon	$z_{i,II}$ gon
1	295	85,511	314,481
2	245	91,678	308,321
3	195	98,005	301,993
4	145	104,368	295,626
5	95	110,640	289,355
6	45	116,720	283,280
NOTE — The series of observations No. 2, 3 and 4 are not printed.			

Table A.4 – Example of field observations for theodolite type B

i	h_i cm	$z_{i,I}$	$z_{i,II}$
1	295	85° 04' 41,0"	274° 56' 41,8"
2	245	86° 58' 33,9"	273° 02' 45,7"
3	195	88° 52' 55,2"	271° 08' 24,7"
4	145	90° 47' 26,3"	269° 13' 52,7"
5	95	92° 41' 48,9"	267° 19' 31,0"
6	45	94° 35' 53,8"	265° 25' 29,7"
NOTE — The series of observations No. 2, 3 and 4 are not printed.			

A.2.2 Results

Results for series of observations No. 1 for theodolite type A (see table A.3):

Approximate values of the parameters x_1 and x_2:

height of tilting axis $x_1^0 = 179$ cm
distance $x_2^0 = 500$ cm

Matrix and vectors of the linearized observation equations:

$$\mathbf{A}=\begin{pmatrix} 12,0821 & 2,8030 & 12,9889 \\ 12,5143 & 1,6519 & 6,3572 \\ 12,7194 & 0,4070 & 1,2467 \\ 12,6738 & -0,8618 & -1,9629 \\ 12,3829 & -2,0803 & -3,1044 \\ 11,8792 & -3,1836 & -2,2504 \end{pmatrix} \quad \mathbf{l}_\mathrm{I}=\begin{pmatrix} 2,3835 \\ 3,3078 \\ 4,1488 \\ 4,5640 \\ 4,3738 \\ 5,0316 \end{pmatrix} \quad \mathbf{l}_\mathrm{II}=\begin{pmatrix} -3,1835 \\ -3,4078 \\ -4,3488 \\ -5,1640 \\ -4,8738 \\ -5,0316 \end{pmatrix}$$

Normal equations:

$12\, o = 2,200$ (o in cgon)

$$\begin{pmatrix} 1838{,}88959 & -29{,}57151 & 324{,}58909 \\ -29{,}57151 & 51{,}91378 & 125{,}46167 \\ 324{,}58909 & 125{,}46167 & 458{,}46870 \end{pmatrix} x = \begin{pmatrix} 616{,}73206 \\ -29{,}50225 \\ 55{,}14601 \end{pmatrix}$$

Unknowns of the adjustment and cc:

$$\begin{aligned} o &= +0{,}1833333 && \text{cgon} \\ dx_1 &= +0{,}3417051 && \text{cm} \\ dx_2 &= -0{,}2352838 && \text{cm} \\ x_3 &= -0{,}0572529 && \text{gon} \end{aligned}$$

$$cc = 215{,}51003\ \text{cgon}^2 - 0{,}40333\ \text{cgon}^2 - 214{,}52462\ \text{cgon}^2 = 0{,}58208\ \text{cgon}^2$$

Result for series No. 1:

$$\begin{aligned} o_1 &= 1{,}8 && \text{mgon} \\ x_{1,1} &= 179{,}34 && \text{cm} \\ x_{2,1} &= 499{,}76 && \text{cm} \\ x_{3,1} &= -57{,}3 && \text{mgon} \\ cc_1 &= 58{,}21 && \text{mgon}^2 \\ s_1 &= 2{,}7 && \text{mgon} \end{aligned}$$

1st series of observations: $s_1 = 2{,}7$ mgon; $o_1 = 1{,}8$ mgon
2nd series of observations: $s_2 = 2{,}1$ mgon; $o_2 = 2{,}0$ mgon
3rd series of observations: $s_3 = 1{,}3$ mgon; $o_3 = 1{,}7$ mgon
4th series of observations: $s_4 = 1{,}8$ mgon; $o_4 = 2{,}2$ mgon

$$s_0 = \sqrt{\frac{16{,}6}{4}}\ \text{mgon} = 2{,}04\ \text{mgon}$$

$$o = \frac{7{,}7}{4}\ \text{mgon} = 1{,}9\ \text{mgon}$$

$$s_{\text{ISO-THEO-V}} = 1{,}44\ \text{mgon}$$

Results for series of observations No. 1 for theodolite type B (see table A.4):

Approximate values of the parameters x_1 and x_2:

height of tilting axis $x_1^0 = 165$ cm
distance $x_2^0 = 1500$ cm

Matrix and vectors of the linearized observation equations:

$$\mathbf{A} = \begin{pmatrix} 2{,}2747 & 0{,}1971 & 1{,}0150 \\ 2{,}2853 & 0{,}1219 & 0{,}5212 \\ 2{,}2909 & 0{,}0458 & 0{,}1559 \\ 2{,}2914 & -0{,}0306 & -0{,}0773 \\ 2{,}2869 & -0{,}1067 & -0{,}1769 \\ 2{,}2773 & -0{,}1822 & -0{,}1431 \end{pmatrix} \quad \mathbf{l}_{\text{I}} = \begin{pmatrix} 1{,}8788 \\ 1{,}7380 \\ 1{,}6658 \\ 1{,}6044 \\ 1{,}5031 \\ 1{,}4614 \end{pmatrix} \quad \mathbf{l}_{\text{II}} = \begin{pmatrix} -0{,}4988 \\ -0{,}4113 \\ -0{,}3341 \\ -0{,}2878 \\ -0{,}1715 \\ -0{,}0697 \end{pmatrix}$$

Normal equations:

$$12\,o = -\,8{,}0783 \quad (o \text{ in } ')$$

$$\begin{pmatrix} 62{,}62320 & 0{,}20558 & 5{,}89902 \\ 0{,}20558 & 0{,}20265 & 0{,}63609 \\ 5{,}89902 & 0{,}63609 & 2{,}76785 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 26{,}55390 \\ 0{,}30667 \\ 3{,}18366 \end{pmatrix}$$

Unknowns of the adjustment and *cc*:

$$\begin{aligned} o &= -0{,}6731916\ ' \\ dx_1 &= +0{,}4173667 \text{ cm} \\ dx_2 &= +0{,}9745900 \text{ cm} \\ x_3 &= +0{,}0367355\ ^\circ \end{aligned}$$

$$cc = 16{,}94121\ (')^2 - 5{,}43824\ (')^2 - 11{,}49855\ (')^2 = 0{,}00442\ (')^2$$

Result for series No. 1:

$$\begin{aligned} o_1 &= -\,0{,}67\ ' \\ x_{1,1} &= 165{,}42 \text{ cm} \\ x_{2,1} &= 1500{,}97 \text{ cm} \\ x_{3,1} &= +\,2{,}2\ ' \\ cc_1 &= 0{,}00442\ (')^2 \\ s_1 &= 1{,}41\ '' \end{aligned}$$

1st series of observations: $s_1 = 1{,}4\ ''$; $o_1 = -\,0{,}67\ '$
2nd series of observations: $s_2 = 1{,}6\ ''$; $o_2 = -\,0{,}69\ '$
3rd series of observations: $s_3 = 1{,}3\ ''$; $o_3 = -\,0{,}65\ '$
4th series of observations: $s_4 = 1{,}9\ ''$; $o_4 = -\,0{,}71\ '$

$$s_0 = \sqrt{\frac{9{,}82''}{4}} = 1{,}57''$$

$$o = \frac{-2{,}72'}{4} = -0{,}68'$$

$$s_{\text{ISO-THEO-V}} = 1{,}44 \text{ mgon}$$

A.2.3 Statistical tests

Example for a test according to question A (theodolite type A):

$s_0 = 1{,}5$ mgon
$1{,}44 \text{ mgon} < 1{,}5 \text{ mgon} \cdot 1{,}20$

Since the above inequality is true, the null hypothesis stating that the standard deviation $s_{ISO-THEO-V}$ = 1,44 mgon is smaller than or equal to the manufacturer's value s = 1,5 mgon is not rejected.

Example for a test according to question B (theodolite type B):

A second surveying team observed four series and obtained s_2 = 2,53" in contrast to the above value s_1 = 1,57".

$$0{,}49 < \frac{(1{,}57'')^2}{(2{,}53'')^2} < 2{,}02$$

$$0{,}49 < 0{,}385 < 2{,}02$$

Since the above inequality is not true, the null hypothesis stating that s_1 and s_2 belong to the same population is rejected.

Example for a test according to question C (theodolite type A):

$$o = 1{,}9 \text{ mgon}$$

$$s_o = \frac{2{,}04 \text{ mgon}}{\sqrt{12}} = 0{,}59 \text{ mgon}$$

$$1{,}9 \text{ mgon} < 0{,}59 \text{ mgon} < 2{,}04$$

$$1{,}9 \text{ mgon} < 1{,}20 \text{ mgon}$$

Since the above inequality is not true, the null hypothesis stating that the index correction o of the vertical circle is equal to zero is rejected.

ANNEX B
(informative)

Example of a computer program suitable for the calculations in accordance with 5.2.4

If you want to use the following C-Program (Source code: ISOVERTI.c, include file MATRIX.h, executable MS-DOS-program ISOVERTI.EXE) you must create with any text-editor an ASCII-file with your measuring data and give it the name ISO.DAT. Then start the program ISOVERTI.EXE. The program will create an output file named ISO.PRN, which you can print or include in any text-system for further word processing. The input file must contain a first line with any comment and a second line with the approximate values for x_1 and x_2. In the third line must be the information whether the following six lines contain angles in centesimal grades (gon) or sexagesimal degrees (°), beginning in column one of the line. The rest of the line is optional. For the data of the theodolite type A the input file looks like:

```
ISO-Example,  Theodolite Type A
 179.0   500.0  Approximations for x1, x2
gon        =  Units of the Vertical Angles
  295.0     85.511     314.481
  245.0     91.678     308.321
  195.0     98.005     301.993
  145.0    104.368     295.626
   95.0    110.640     289.355
   45.0    116.720     283.280
```

and for the theodolite of type B the input-file looks like:

```
ISO-Example, Theodolite Type B   / Sexagesimal
 165.0  1500.0  Approximations for x1, x2
degrees    =  units of the vertical angles   (° ' ")
 295.0     85 04 41.0     274 56 41.8
 245.0     86 58 33.9     273 02 45.7
 195.0     88 52 55.2     271 08 24.7
 145.0     90 47 26.3     269 13 52.7
  95.0     92 41 48.9     267 19 31.0
  45.0     94 35 53.8     265 25 29.7
```

In order to get more flexibility for several data-files with your own name convention, you only should give them the name extension .DAT and use a batch file to copy the appropriate files to and from the standard names required and supplied by ISOVERTI.EXE. As an example take the batch file VERTICAL.BAT, which accepts the name of your desired datafile as command line parameter.

Contents of VERTICAL.BAT:

```
copy %1.dat iso.dat
isoverti.exe
copy iso.prn %1.prn
```

If your input file has the name ISO_B.DAT, you will start the batch file with

```
VERTICAL ISO_B
```

If no errors occur during the run, you will find a new file with the name ISO_B.PRN, which contains your results.

Contents of the source file MATRIX.h:

```
/****  Include-File with the needed Matrix Functions ****/
int index (int n, int i, int j)
{ return n*(i-1)+j;}

void ShowMatrix(char Text[], double N[], int n, int m)
{  int i,j;
   puts(Text);
   fputs(Text,output);
   fprintf(output,"\n");
   for (i=1;i<=n;++i)
   {  for (j=1;j<=m;++j)
      {  printf("%15.5lf",N[index(m,i,j)]);
        fprintf(output,"%15.5lf",N[index(m,i,j)]);
      }
       printf("\n");
      fprintf(output,"\n");
   }
} /* End of ShowMatrix */

void InverseMatrix (int n, double A[])
{  int i,j,k;
   for (k=1;k<=n;++k)
   {  for (j=1; j<=n; ++j)
      {  if (k!=j)
         {  A[index(n,k,j)]=-A[index(n,k,j)]/A[index(n,k,k)];
            for (i=1;i<=n;++i)
            {  if (k!=i)

A[index(n,i,j)]=A[index(n,i,j)]+A[index(n,k,j)]*A[index(3,i,k)]
;
            }
         }
      }
      A[index(n,k,k)]=1/A[index(n,k,k)];
      for (i=1;i<=n;++i)
                              if    (k!=i)    A[index(n,i,k)]    =
A[index(n,i,k)]*A[index(n,k,k)];
   }
} /* End of InverseMatrix */

void TransMatrixTimesMatrix (double A[], int ma, int na,
                             double B[], int mb, int nb,
                             double P[], int mp, int np)
{
   int i,j,k;
   double s;
   for (i=1;i<=na; ++i)
   {  for (j=1;j<=nb;++j)
      {   s = 0;
                                 for    (k=1;k<=ma;++k)    s    =
s+A[index(na,k,i)]*B[index(nb,k,j)];
```

```
                    P[index(np,i,j)] = s;
                }
             }
} /* End of TransMatrixTimesMatrix */

void MatrixSubtrMatrix (double A[], int ma, int na,
                        double B[], int mb, int nb,
                        double D[], int md, int nd)
{
   int i,j,k;
   double s;
   for (i=1;i<=ma; ++i)
     for (j=1;j<=nb;++j)
          D[index(nd,i,j)] = A[index(na,i,j)]-B[index(nb,i,j)];
} /* End of TransMatrixTimesMatrix */
```

Contents of source-file ISOVERTI.c:

```
#include        <math.h>
#include        <stdio.h>
#include        <string.h>

char project[]="ISO";
char inputfile[13];
char outputfile[13];
FILE *input, *output;


#include        "matrix.h"


/*** Function Prototypes: ***/
int index (int n, int i, int j);
double round (double x, int n);

void ShowMatrix(char Text[], double N[], int n, int m);
void InverseMatrix (int n, double A[]);
void TransMatrixTimesMatrix (double A[], int ma, int na,
                             double B[], int mb, int nb,
                             double P[], int mp, int np);
void MatrixSubtrMatrix (double A[], int ma, int na,
                        double B[], int mb, int nb,
                        double D[], int md, int nd);

struct angle ZenithAngle (int face,double h,
                    double x0, double x1, double x2, double x3);
void lowcase (char string[]);
void sexatodec(struct angle *z);
void ReadFile (void);
void ObservationEquations (void);
void NormalEquations (double A[],double l_I[], double l_II[],
                      double N[], double r[]);
void Solution  (double N[], double r[], double x[], double *sk);

/*** Global variables: ***/
```

```
char units[5];
char unitsfa[5];
int degree;
double piUnit;
        double pi1_2;
        double pi3_2;
        double pi;
        double fa;
        double rho;
        double h[7];
        struct angle
        {   double z;
            int deg;
            int min;
            double sec;
        }; /* End of struct angle */
        struct angle zI[7],zII[7],za;
        double x10,x20;
        char unit[6];
        char comment[80];
        double A[190];
        double l_I[7],l_II[7];
        double Diff[7];
        double N[10];
        double r[4];
        double x[4];


        double round (double x, int n)
        {   x= x*pow(10,n);
            if (x>0) x = x+0.5;
            if (x<0) x = x-0.5;
            return (x-fmod(x,1.0))/pow(10,n);
        }   /* End of round */

        struct angle ZenithAngle (int face,double h, double x0, double
        x1, double x2, double x3)
        {   double aux;  struct angle za;

            if (face==1)
                                  aux    =    rho*(pi1_2-atan((h*cos(x3/rho)-
        x1)/(x2+h*sin(x3/rho))))-x0;
            else
                                  aux    =    rho*(pi3_2+atan((h*cos(x3/rho)-
        x1)/(x2+h*sin(x3/rho))))-x0;
            za.z= aux;
            za.deg = (int)(aux - fmod(aux,1.0));
            aux = (aux-za.deg)*60;
            za.min = (int)(aux - fmod(aux,1.0));
            aux = (aux-za.min)*60;
            za.sec = aux;
            return za;
        } /* End of ZenithAngle */

        void lowcase (char string[])
        {   int i;
            for (i=0;string[i];++i)
```

```
              if (string[i]<='Z') string[i]=string[i]+32;
        } /* End of lowcase */

        void sexatodec(struct angle *z)
        {
           (*z).z=(*z).deg+((*z).sec+(*z).min*60)/3600;
        }

void ReadFile (void)
{  int i,n;
   char lineRest[80];
   pi=4.0*atan(1.0);
   pi1_2=pi/2;
   pi3_2=pi*1.5;

   inputfile[0]='\0';
   outputfile[0]='\0';
   strcat(inputfile,project);
   strcat(outputfile,project);
   strcat(inputfile,".DAT");
   strcat(outputfile,".PRN");

   output= fopen (outputfile,"w");
   input = fopen (inputfile,"r");

   printf("Accuracy of Vertical Angles\n\n");
  fprintf(output,"Accuracy of Vertical Angles\n\n");
   fgets (comment,80,input);
   printf ("%s",comment);
  fprintf (output,"%s",comment);

   fscanf(input,"%lf %lf ", &x10,&x20);
   fgets (lineRest,80,input);
   printf("\nx1= %7.1lf cm ; x2= %7.1lf cm  ; ",x10,x20);
   printf("%s",lineRest);
    fprintf  (output,"\nx1=  %7.1lf  cm  ;  x2=  %7.1lf  cm    ;
",x10,x20);
  fprintf (output,"%s",lineRest);

   fgets (units,4,input);
   lowcase (units);
   fgets (lineRest,80,input);
   degree= !strncmp (units,"deg",3);
   if (degree)
   {
      piUnit=180; /* Degrees */
       fa= 60; /* factor for angles  ; for calculations using
minutes */
      units[0]='\0'; strcat(units,"°");
      unitsfa[0]='\0'; strcat(unitsfa,"'");
   }
   else
   {
      piUnit=200; /* Gon     */
       fa=100; /* factor for angles    ; for calculations using
cgon*/
      units[0]='\0'; strcat(units,"gon");
```