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TECHNICAL REPORT

colour inside

Transmission properties of cascaded two-ports or quadripols - Background of terms and definitions

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

BACKGROUND OF TERMS AND DEFINITIONS OF CASCADED TWO-PORTS

FOREWORD

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IEC 62152, which is a technical report, has been prepared by IEC technical committee 46: Cables, wires, waveguides, r.f. connectors, r.f. and microwave passive components and accessories.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
46/283/DTR	46/300/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This second edition cancels and replaces the first edition published in 2004 and constitutes some technical improvements.

Important terms and definitions have been added.

Some of the terms are better described in the German language and also many countries have originally taken terms and definitions from German and translated them into their own language.

Therefore important terms have been added in German in the form of a footnote.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the maintenance result date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual edition of this document may be issued at a later date.

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BACKGROUND OF TERMS AND DEFINITIONS OF CASCADED TWO-PORTS

1 Scope

It is important and practical that components of a transmission chain can be separated and tested separately. To accomplish this, well-defined interfaces and measuring techniques, including agreed terms and definitions, are required.

This technical report has two main goals. It lays the foundation for agreement on the fundamental terms and definitions to be used world-wide in describing the transmission properties of a two-port or quadripole. The report builds a bridge between the classical quadripole theory and the scattering matrix presentation which is based on incident and reflecting square root of power waves at the input and output of a two-port. Finally, it is shown that the two concepts are bound together through simple equations and are fundamentally identical.

The quadripole theory was originally developed for voice- and carrier-frequency technologies and transmission, and later for microwaves, but both can be used through the whole frequency range.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-726, *International Electrotechnical Vocabulary – Chapter 726: Transmission lines and waveguides*

IEC 61156-1, *Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification*

IEC/TR 61156-1-2, *Multicore and symmetrical pair/quad cables for digital communications – Part 1-2: Electrical transmission characteristics and test methods of symmetrical pair/quad cables used for digital communications*

3 Terms and definitions, symbols, units and abbreviated terms

For the purposes of this document, the terms and definitions given in IEC 60050-726, IEC 61156-1, IEC/TR 61156-1-2, as well as the following defintions, apply.

3.1 Terms and definitions

3.1.1
complex operational attenuation[1]
quotient of the unreflected square root of the power wave fed into the reference impedance R_1 of the input of the two-port and the square root of the power wave consumed by the load R_2 of the two-port expressed in dB and radians

[1] Komplexe Betriebs-Dämpfung.

NOTE By defining a new quantity, operational insertion loss, which is the same as the operational attenuation when the reference impedances on both sides of the two-port are the same $R_1 = R_2$, the problem of insertion loss and operational attenuation is solved in most usual cases.

3.1.2

complex operational insertion loss[2]

quotient of the unreflected square root of the power wave fed into the reference impedance R_1 of the measurement system and the square root of the power wave consumed by the load R_1 of the two-port expressed in dB and radians

NOTE In the IEV, insertion loss is understood as the loss produced by inserting a two-port into a separated point of the transmission chain. Because of varying impedances along the transmission line, it leads to deviation in the overall losses depending on where in the chain the two-port is inserted. This is called insertion loss deviation (ILD). In “complex operational insertion loss” the reference impedances at both sides of the two-port are equal.

3.2 Symbols, units and abbreviated terms

3.2.1 Two-port electrical symbols, units and related terms

E_0 generator source voltage (V)

R_1, R_2 reference impedance at the two-port input and output, respectively (Ω)

R reference impedance at the two-port input and output, respectively (Ω)

U_1, U_2 voltage at the two-port input and output, respectively (V)

U_0 voltage at the reference impedance for the condition of matched generator reference impedance (V)

Z_{01}, Z_{02} complex characteristic impedance at the two-port input and output, respectively

$\sqrt{P_2}$ square root of power wave from the two-port (W½)

$\sqrt{P_0}$ unreflected square root of power wave from the generator for the condition of matched generator reference impedance (W½)

$\sqrt{P_{02}}$ reflected square root of power wave coming from the reference impedance at the two-port output (W½)

T_B operational transfer function

T image transfer function

T'_B insertion transfer function

S_{21} forward transfer scattering parameter

Γ_B complex operational attenuation

A_B real part of Γ_B and is the operational attenuation

$$A_B = -20 \times \log_{10} |S_{21}| \text{ (dB) or}$$

$$A_B = -\ln |S_{21}| \text{ (Np)}$$

B_B imaginary part of Γ_B and is the operational attenuation phase shift

$$= -\arg(S_{21}) \text{ (rad)}$$

Γ'_B complex insertion attenuation or loss

A'_B real part of Γ'_B (dB) or (Np)

B'_B imaginary part of Γ'_B (rad)

Γ complex image attenuation

---

2 Komplexe Betriebs-Einfüge-Dämpfung.

A real part of Γ (dB) or (Np)

B imaginary part of Γ (rad)

j imaginary denominator

arg argument operator of a complex number

Z_C, Z_0 complex characteristic impedance, or mean characteristic impedance if the pair is homogeneous or free of structure (also used to represent a function fitted result) (Ω)

Z_{CN} nominal characteristic impedance and resistive part of the mean characteristic impedance Z_C value at a given frequency with tolerance at a given frequency (Ω)

Z_N nominal impedance of the link and/or terminals (the system) between which the two-port is operating (Ω)

Z_R (nominal) reference impedance used in measurements, normally, $Z_R = Z_N$; (Ω)

RL complex operational return loss (dB)

ρ_B reflection coefficient

SRL structural return loss (dB)

Z_W measured input image impedance (Ω)

Re real part operator for a complex variable

Im imaginary part operator for a complex variable

R pair resistance (Ω/m)

L pair inductance (H/m)

L_∞ pair inductance asymptotic value at high frequencies (H/m)

G pair conductance (S/m)

C pair capacitance (F/m)

v_P phase velocity of cable (m/s)

ω radian frequency (rad/s)

l length (m)

Δf frequency difference between input impedance minima of a short-circuited transmission line (MHz)

S, ρ complex reflection coefficient of the junction

$\sqrt{P_r}$ reflected square root of power wave at the junction ($W^{1/2}$)

$\sqrt{P_i}$ incident square root of power wave at the junction ($W^{1/2}$)

Z_1, Z_2 line impedance to the left and right of the junction, respectively (Ω)

U_i, U_r incident and reflected voltage at the junction, respectively (V)

V_i, V_r incident and reflected voltage at the junction, respectively (V)

I_i, I_r incident and reflected current at the junction, respectively (A)

Γ_s complex reflection loss at the junction

A_s reflection loss

$$A_s = 20 \times \log_{10} \left| \frac{z_N + 1}{2 \times \sqrt{z_N}} \right| \text{ (dB)}$$

A_r return loss

$$A_r = 20 \times \log_{10} \left| \frac{z_N + 1}{z_N - 1} \right| \text{ (dB)}$$

z_N normalized impedance given by $z_N = \frac{Z_2}{Z_1} = r + jx$

r x-axis ordinate

x y-axis ordinate

Γ_m mismatch loss of a junction (not recommended)

3.2.2 Transmission line equation electrical symbols and related terms

α attenuation coefficient (Np/m)

β phase coefficient (rad/m)

γ propagation coefficient (Np/m, rad/m)

v_P phase velocity of cable (m/s)

v_G group velocity of cable (m/s)

τ_P phase delay time (s/m)

τ_G group delay time (s/m)

Z_C complex characteristic impedance, or mean characteristic impedance if the pair is homogeneous or free of structure (also used to represent a function fitted result) (Ω)

$\angle Z_C$ angle of the characteristic impedance in radians

Z_∞ high frequency asymptotic value of the characteristic impedance (Ω)

l length (m)

ω radian frequency (rad/s)

f frequency (Hz)

R' first derivative of R with respect to ω

C' first derivative of C with respect to ω

L' first derivative of L with respect to ω

R_0 d.c. resistance of a round solid wire with radius r (Ω/m)

R_C constant with frequency component of resistance which is about one-quarter of the d.c. resistance (Ω/m)

R_S square-root of frequency component of resistance (Ω/m)

L_E external (free space) inductance (H/m)

L_I internal inductance whose reactance equals the surface resistance at high frequencies (H/m)

σ specific conductivity of the wire material (S/m)

ρ resistivity of the wire material (Ω/m^2)

μ permeability of the wire material (H/m)

r radius of the wire (m)

δ skin depth (not to be confused with the dissipation factor tan δ) (m)

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

tan δ dissipation factor

tan $\delta = G/(\omega C)$

q forward echo coefficient at the far end of the cable at a resonant frequency

p reflection coefficient measured from the near end of the cable at a resonant frequency, $p = 10^{-PSRL/20} = \left| \frac{Z_{CM} - Z_C}{Z_{CM} + Z_C} \right|$

A_Q forward echo attenuation at a resonant frequency (dB)

$A_Q = -20 \times \log_{10} |q|$

$PSRL$ structural return loss at a resonant frequency (dB),

$PSRL = -20 \times \log_{10} |p|$

K $= 2 \times \alpha l - 1$ when $2 \times \alpha l$ >> 1 (Np)

A_Q $= 2 \times PSRL - 20 \times \log_{10}(2 \times \alpha l - 1)$ (dB) where $2 \times \alpha l$ is in Np

Z_{OC} complex measured open circuit impedance (Ω)

Z_{SC} complex measured short circuit impedance (Ω)

Z_{CM} characteristic impedance as measured (with structure) (Ω)

$Z_{CM} = \sqrt{Z_{SC} Z_{OC}}$

Z_{MEAS} complex measured impedance (open or short) (Ω)

Z_{IN} input impedance of the cable when it is terminated by Z_L (Ω)

Z_{OUT} output impedance of the cable when the input of the cable is terminated by Z_G (Ω)

Z_T terminated impedance measurement made with the opposite end of the cable pair terminated in the reference impedance Z_R (Ω)

ς reflection coefficient measured in the terminated measurement method $\varsigma = \frac{Z_R - Z_C}{Z_R + Z_C}$

Z_G termination at the cable input when defining the output impedance of the cable Z_{OUT} (Ω)

Z_L termination at the cable output when defining the input impedance of the cable Z_{IN} (Ω)

L_0, L_1, L_2, L_3 least squares fit coefficients for angle of the characteristic impedance

K_0, K_1, K_2, K_3 least squares fit coefficients of the characteristic impedance

$|Z_C|$ fitted magnitude of the characteristic impedance (Ω)

$|Z_{CM}|$ measured magnitude of the characteristic impedance (Ω)

$\angle(V_{1N})$ input angle relative to a reference angle in radians

$\angle(V_{1F})$ output angle relative to the same reference angle in radians

k multiple of 2π radians;

S_{11} reflection coefficient measured with an S parameter test set

4 Transfer functions and complex attenuations or losses of a two-port

4.1 General remarks

Figure 1 indicates the variables and their relationships for defining the transfer functions of a two-port. E_0 is the generator source voltage in Figure 1.

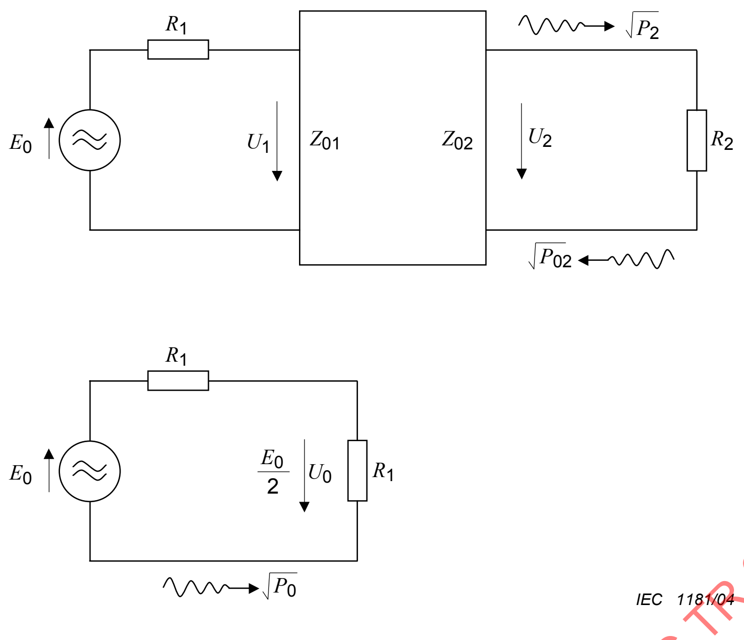


Figure 1 – Defining the transfer functions of a two-port

4.2 Operational transfer function (T_B)

Referring to Figure 1, the operational transfer function T_B is defined as the ratio of the square root of the power wave into the load (equal to reference impedance R_2) of a two-port $\sqrt{P_2}$ with the unreflected square root of power wave $\sqrt{P_0}$ from the generator with a source impedance equal to the reference impedance R_1. See Equation (1).

$$T_B = \frac{\sqrt{P_2}}{\sqrt{P_0}} = \frac{U_2/\sqrt{R_2}}{U_0/\sqrt{R_1}} = S_{21} = \left.\frac{\sqrt{P_2}}{\sqrt{P_0}}\right|_{\sqrt{P_{02}}=0} \tag{1}$$

4.2.1 Image transfer function (T)

The operational transfer function becomes the image transfer function T when the reference impedance becomes equal to the input and output characteristic impedances Z_{01} and Z_{02} of the two-port.

4.2.2 Insertion transfer function (T'_B)

The operational transfer function becomes the insertion transfer function T'_B when $R_1 = R_2 = R$.

4.3 Complex attenuation

4.3.1 Complex operational attenuation (Γ_B)[3]

The complex operational attenuation is given by Equation (2):

3 Komplexe Betriebs-Dämpfung.

$$\Gamma_{\mathrm{B}} = A_{\mathrm{B}} + \mathrm{j} \cdot B_{\mathrm{B}} = \ln \frac{1}{T_{\mathrm{B}}} = -20 \times \log_{10} |T_{\mathrm{B}}| - \mathrm{j} \cdot \arg(T_{\mathrm{B}}) \qquad (2)$$

4.3.2 Complex image attenuation (Γ)[4]

The complex image attenuation is given by Equation (3):

$$\Gamma = A + \mathrm{j} \cdot B = \ln \frac{1}{T} = -20 \times \log_{10} |T| - \mathrm{j} \cdot \arg(T) \qquad (3)$$

4.3.3 Complex insertion attenuation or loss (Γ'_{B})[5]

The complex insertion attenuation or loss is given by Equation (4):

$$\Gamma'_{\mathrm{B}}\Big|_{R1=R2=R} = A'_{\mathrm{B}} + \mathrm{j} \cdot B'_{\mathrm{B}} = \ln \frac{1}{T'_{\mathrm{B}}} = -20 \times \log_{10} |T'_{\mathrm{B}}| - \mathrm{j} \cdot \arg(T'_{\mathrm{B}}) \qquad (4)$$

4 Komplexe Wellen-Dämpfung.

5 Komplexe Einfüge-Dämfung.

Annex A
(normative)

Concepts of normalized voltage waves, square root of power waves and operational attenuation and losses

A.1 General

It is advantageous to operate, by the square root of a reference impedance (normally application impedance of the system), with normalized voltage waves corresponding to the square root of power waves.

In this way the scattering parameters are defined. For example, S_{21} is the forward operational transfer function and S_{11} is the operational reflection coefficient.

Two primary reasons for using the square root of the impedance normalized voltage waves or the square root of the power waves are

a) that the network analyser is measuring voltages, and

b) because the natural logarithm, ln, of a complex quantity $z = x + \mathrm{j} \cdot y = |z| \cdot \mathrm{e}^{\mathrm{j} \cdot \arg(z)}$ is directly $\ln(z) = \ln|z| + \mathrm{j} \cdot \arg(z)$ and $\ln|z|$, in nepers, can be expressed in decibels $20 \times \log_{10}|z|$ and the imaginary part still remains arg(z) in radians, as, for example,

$$\Gamma_\mathrm{B} = A_\mathrm{B} + \mathrm{j} \times B_\mathrm{B} = -20 \times \log_{10}|S_{21}| - \mathrm{j} \times \arg(S_{21})$$

(see Equations (A.1) and (A.2)).

Furthermore, usage of operational quantities means the measurements are always made between resistive terminations in well-defined circumstances.

This means that the impedances at a reference plane between the cascaded units of the system are specified.

Individual units can be specified and tested separately and made by different manufacturers.

This makes open systems, networks and cabling possible.

A.2 Complex operational attenuation or operational propagation coefficient (Γ_B)

The complex operational attenuation (complex operational loss) introduced by a two-port component, cascade of components, link, cable assembly, etc. into a system is defined by using the scattering parameter S_{21} as

$$\Gamma_\mathrm{B} = A_\mathrm{B} + \mathrm{j} \cdot B_\mathrm{B} = \ln(1/S_{21}) = -\ln|S_{21}| - \mathrm{j} \cdot \arg(S_{21}) \quad \text{(A.1)}$$

$$\Gamma_\mathrm{B} = A_\mathrm{B} + \mathrm{j} \cdot B_\mathrm{B} = -20 \times \log_{10}|S_{21}| - \mathrm{j} \cdot \arg(S_{21}) \quad \text{(A.2)}$$

NOTE 1 A_B is equal to the ratio of the unreflected complex power (voltage × current) sent into a two-port, to the complex power consumed by the load of the two-port, in decibels. The load is normally a resistance equal to the application impedance of the system Z_N. When the generator and load impedances are the same, complex operational attenuation becomes complex operational insertion loss.

NOTE 2 From the theory of complex functions:

$$\ln z = \ln|z| + \mathrm{j} \cdot \arg(z)$$

where

$$z = x + \mathrm{j} \cdot y = |z| \cdot \mathrm{e}^{\mathrm{j} \cdot \arg(z)}$$

and, by using the square root of power waves, we can write, for the natural logarithms of the ratio of two square root of complex power waves:

$$\ln \frac{\sqrt{P_1}}{\sqrt{P_2}} = \ln\left|\frac{\sqrt{P_1}}{\sqrt{P_2}}\right| + \mathrm{j} \cdot \arg\left(\frac{\sqrt{P_1}}{\sqrt{P_2}}\right) = \Gamma = A + \mathrm{j} \cdot B$$

where A is in nepers and B in radians.

When A is expressed in decibels, B will not be affected; it remains in radians.

A.3 Impedances

The different kinds of impedances are defined as follows:

a) the nominal characteristic impedance[6] Z_{CN} (of a two-port) is the resistive part of the mean characteristic impedance Z_C specified with a tolerance at a given frequency;

b) Z_N is the nominal impedance of the system terminals between which the two-port is operating;

c) Z_R is the (nominal) reference impedance used in measurements, normally $Z_R = Z_N$.

A.4 Operational reflection coefficient (S_{11})

The operational reflection coefficient of a two-port is equal to the scattering parameter S_{11} of the two-port. It equals the reflection coefficient ρ_B at the input when the two-port is terminated with its reference impedances Z_{R1} normally equal to the nominal impedance of the system terminals.

$$S_{11} = \rho_B = \frac{Z_{in} - Z_{R1}}{Z_{in} + Z_{R1}} \qquad \text{(A.3)}$$

A.5 Return loss

A.5.1 Complex operational return loss of a transmission line (RL_B)

The complex operational return loss, RL_B of a transmission line is given in Equation (A.4):

6 Nominale Wellen-Widerstand.

$$RL_B = \ln\frac{1}{\rho_B} = -\ln(\rho_B) = -\ln|\rho_B| - j\cdot\arg(\rho_B)$$
$$= -20\times\log_{10}|\rho_B| - j\cdot\arg(\rho_B) \quad (A.4)$$

A.5.2 Structural return loss of a transmission line (SRL)

SRL is the return loss where the mismatch effects at the input and output of transmission line have been eliminated (compare with the continuous wave (CW) burst measurement method). This quantity is obtained by calculation (Equation (A.5)) using the measured input image input impedance[7] and measured, calculated (Equation (A.6)) and curve fitted mean characteristic impedance[8] where both are complex quantities. The structural return loss is as follows:

$$SRL = -20\times\log_{10}\left|\frac{Z_W - Z_0}{Z_W + Z_0}\right| - j\cdot\arg\left(\frac{Z_W - Z_0}{Z_W + Z_0}\right) \quad (A.5)$$

See Clauses A.6 and A.7.

The complex characteristic impedance[9] of a homogeneous transmission line is as follows:

$$Z_0 = \mathrm{Re}(Z_0) + j\cdot\mathrm{Im}(Z_0) \approx \sqrt{\frac{L_\infty}{C}}\left(1+\frac{R}{2\omega L}\right) - j\cdot\sqrt{\frac{L_\infty}{C}}\,\frac{R}{2\omega L} \quad (A.6)$$

$$\mathrm{Re}(Z_0) \approx \sqrt{\frac{L_\infty}{C}}\left(1+\frac{R}{2\omega L}\right) \approx \frac{1}{v_p C} = \frac{1}{2\Delta f\, l\, C} \quad (A.7)$$

$$-\mathrm{Im}(Z_0) \approx \sqrt{\frac{L_\infty}{C}}\left(\frac{R}{2\omega L}\right) = \mathrm{Re}(Z_0) - \sqrt{\frac{L_\infty}{C}} \quad (A.8)$$

where v_p and l are the phase velocity and length of the transmission line and C the capacitance of the low dielectric loss line measured at such a low frequency that the length is electrically short, $l < \lambda/40$. Δf is the distance in frequency between two input impedance minima of the short-circuited measured transmission line; and L_∞ the asymptotic value of the inductance reached at high frequencies. See Annex B.

NOTE It is important to distinguish between the two return losses RL and SRL although they are normally not measured separately.

A.5.3 Reflection loss of a junction

The quantities that determine the reflection loss of a junction are shown in Figure A.1. Normalized plots of reflection loss and return loss are given in Figure A.2.

$$\sqrt{P_i} \rightarrow$$
$$\leftarrow \sqrt{P_r}$$

7 Eingangs- Wellen-Widerstand, Z_W.

8 Mitlerer Wellen-Wiederstand, Z_0.

9 Komplexe Wellen-Wiederstand.

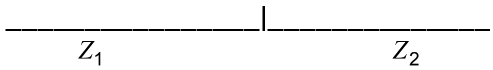


Figure A.1 – Reflection at a junction

$$S = \rho = \sqrt{\frac{P_r}{P_i}} = \frac{V_r}{V_i} = \frac{U_r}{U_i} = -\frac{I_r}{I_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad \text{(A.9)}$$

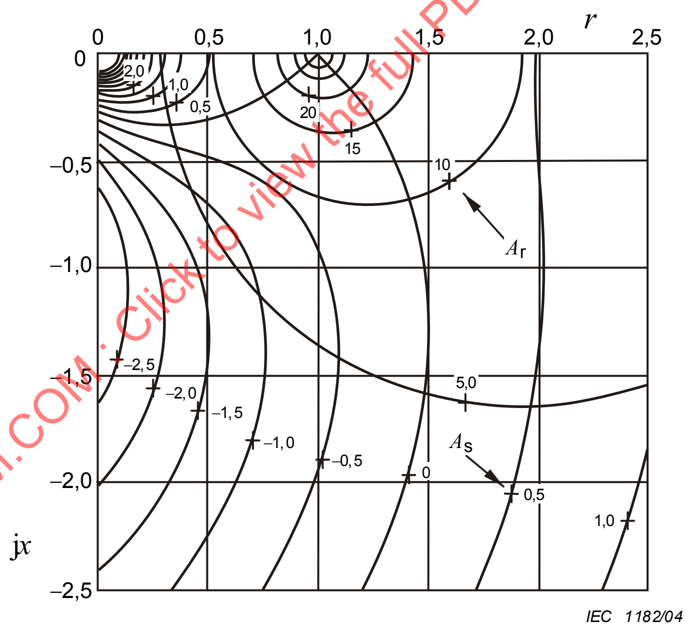
S is the complex reflection coefficient of the junction. See Annex C.

$$\Gamma_S = -\ln\sqrt{\left(1-S^2\right)} = -\ln\left|\sqrt{\left(1-S^2\right)}\right| - j\cdot\arg\left(\sqrt{\left(1-S^2\right)}\right) \qquad \text{(A.10)}$$

or

$$\Gamma_S = -\ln\sqrt{\left(1-S^2\right)} = -20\times\log_{10}\left|\sqrt{\left(1-S^2\right)}\right| - j\cdot\arg\left(\sqrt{\left(1-S^2\right)}\right) \qquad \text{(A.11)}$$

$$\Gamma_S = -\ln\sqrt{\left(1-S^2\right)} = -10\times\log_{10}\left|\left(1-S^2\right)\right| - j\times\frac{1}{2}\times\arg\left(1-S^2\right) \qquad \text{(A.12)}$$



Key

A_s is the reflection loss given by $A_s = 20\times\log_{10}\left|\frac{z_N + 1}{2\times\sqrt{z_N}}\right|$ expressed in dB;

A_r is the return loss given by $A_r = 20\times\log_{10}\left|\frac{z_N + 1}{z_N - 1}\right|$ expressed in dB;

z_N is the normalized impedance given by $z_N = \frac{Z_2}{Z_1} = r + jx$ (see Figure A.1);

r is the x-axis ordinate;
x is the y-axis ordinate.

Figure A.2 – Constant value A_s and A_r curves on a complex plane $z = x + jy$

A.5.4 Mismatch loss of a junction (Γ_m)(not recommended)

Mismatch loss of a junction is expressed as a function of S the complex reflection coefficient ,as follows:

$$\Gamma_m = -\ln\sqrt{\left(1-|S|^2\right)} = -\ln\left|\sqrt{\left(1-|S|^2\right)}\right| - j\cdot \arg\left(\sqrt{\left(1-|S|^2\right)}\right) \quad \text{(A.13)}$$

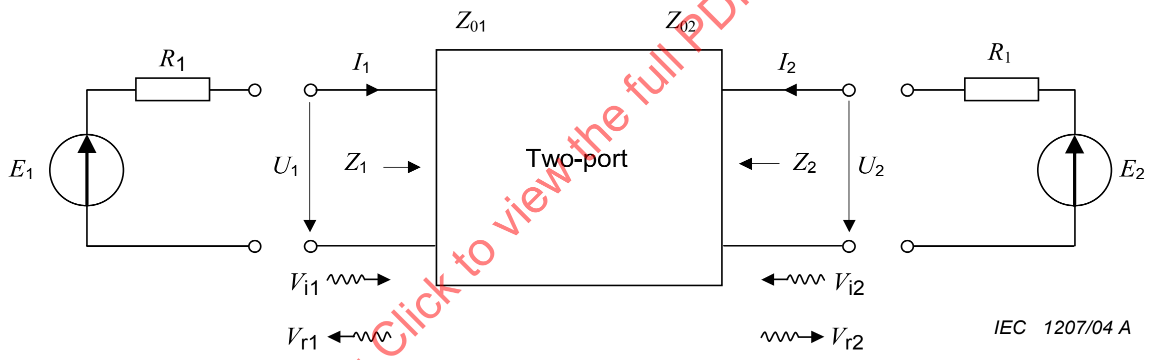
or

$$\Gamma_m = -\ln\sqrt{\left(1-|S|^2\right)} = -20\times\log_{10}\left|\sqrt{\left(1-|S|^2\right)}\right| - j\cdot \arg\left(\sqrt{\left(1-|S|^2\right)}\right) \quad \text{(A.14)}$$

$$\Gamma_m = -\ln\sqrt{\left(1-|S|^2\right)} = -10\times\log_{10}\left|\left(1-|S|^2\right)\right| - j\times\frac{1}{2}\times\arg\left(1-|S|^2\right) \quad \text{(A.15)}$$

A.6 Definition of the characteristic input impedance of a transmission line (cable pair)

The important variables associated with the two-port representation of a transmission line are given in Figure A.3. V_i and V_r are the incident and reflected square root of power waves.



Key

E_1, E_2	network analyser at input, output, respectively	V_{i1}, V_{i2}	incident square root of power waves at input and output, respectively
R_1	reference impedance at input and output	V_{r1}, V_{r2}	reflected square root of power waves at input and output, respectively
I_1, I_2	current at input and output, respectively	Z_1, Z_2	impedance at input and output, respectively
U_1, U_2	voltage at input and output, respectively	Z_{01}, Z_{02}	characteristic input impedance or complex image input impedance[10]

Figure A.3 – Two-port representation of a transmission line

The characteristic input impedance or complex image input impedance[11] is given by Equation (A.16):

$$Z_{01} = R_1\frac{1+S_{W11}}{1-S_{W11}} = \sqrt{Z_{\text{shortc}}\cdot Z_{\text{openc}}} \quad \text{(A.16)}$$

10 Komplexe eingangs-wellenwiderstand.

where

S_{W11} is the the complex image reflection factor at the input when there are no reflections from the far-end, $V_{i2} = 0$ (see note);

Z_{shortc} is the the measured impedance at the input when the output is terminated in a short-circuit;

Z_{openc} is the the measured impedance at the input when the output is terminated in an open circuit.

The complex image reflection factor[11] at the input may also be expressed by Equation (A.17):

$$\rho_{W11} = S_{W11} = \frac{Z_{01} - R_1}{Z_{01} + R_1} \tag{A.17}$$

where

$\rho_{W11} = S_{W11}$ is the complex image reflection factor at the input when there are no reflections from the far-end, ($V_{i2} = 0$) (see note).

NOTE The condition $V_{i2} = 0$ can be simulated by a long line terminated with its nominal impedance. When the roundtrip attenuation of the line added with the return loss at the far end is not less than 40 dB (see Equation (A.18)) the maximum uncertainty in Z_{01} is less than 2 %. With more than 60 dB round-trip attenuation the maximum uncertainty in Z_{01} is less than 0,2 %.

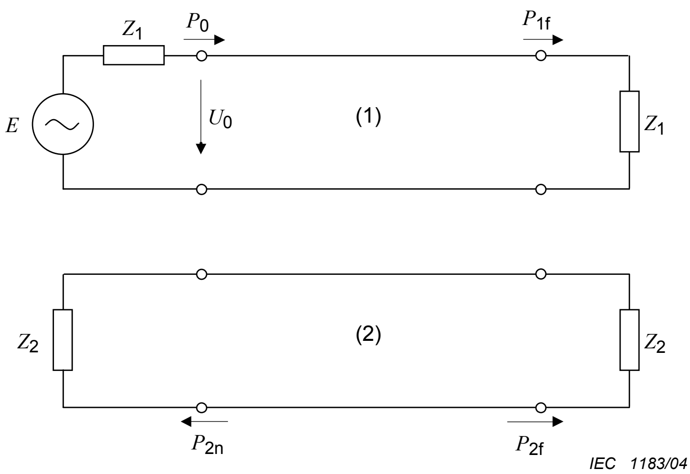
$$\left(2 \times \alpha \cdot L - 20 \times \log_{10}\left|\rho_{W22}\right|\right) \geq 40 \tag{A.18}$$

A.7 General coupling transfer function, crosstalk and echoes

A.7.1 General

Figure A.4 indicates the key variables and their relationships for defining the coupling transfer function between two systems.

11 Komplexe Eingangs-Wellen-Reflexions-Faktor.



Key

(1), (2)	disturbing and disturbed systems, respectively	P_0	unreflected power sent into the near end of the system (1)
E	generator source voltage in system (1)	U_0	input voltage, system (1)
Z_1, Z_2	terminations	P_{1f}, P_{2n}, P_{2f}	power in systems (1) and (2)

Figure A.4 – Coupling between two systems

Coupling transfer functions T_n and T_f may be defined at the near and far ends, respectively (see Equation (A.5)).

$$T_{n,f} = \frac{\sqrt{P_{2n,f}}}{\sqrt{P_0}} = \frac{U_{2n,f}\Big/\sqrt{Z_{2n,f}}}{U_0\Big/\sqrt{Z_1}} = \frac{\sqrt{Z_1}}{U_0}\,\frac{U_{2n,f}}{\sqrt{Z_{2n,f}}} \tag{A.5}$$

where

$T_{n,f}$ is the complex coupling transfer function at near or far end;

n, f are the near end and far end, respectively;

$U_{2n,f}$ is the voltage at the near or far end of system (2);

$Z_{2n,f}$ is the input impedance at the near or far end of system (2).

The coupling transfer function is a general term that is valid through the whole frequency range.

A.7.2 Transfer function for near-end and far-end crosstalk

This may be expressed in decibels and radians, e.g. near-end and far-end crosstalk attenuation (*NEXT* and *FEXT*) as given in Equation (A.6).

$$T_{n,f} = 20 \times \log_{10}\left|\frac{\sqrt{P_{2n,f}}}{\sqrt{P_0}}\right| + j \cdot \arg\left(\frac{\sqrt{P_{2n,f}}}{\sqrt{P_0}}\right) \tag{A.6}$$

where

$20 \times \log_{10}\left|\frac{\sqrt{P_{2n,f}}}{\sqrt{P_0}}\right|$ is expressed in decibels;

$j \cdot \arg\left(\frac{\sqrt{P_{2n,f}}}{\sqrt{P_0}}\right)$ is expressed in radians.

A.7.3 Complex operational transfer attenuation

The complex operational transfer coupling function may be expressed for screening, unbalance or crosstalk attenuations. See Equation (A.7).

$$\Gamma_x = A_x + jB_x = -20 \log_{10}|T_x| - j \arg(T_x) \qquad (A.7)$$

where

Γ_x is the complex operational attenuation;

A_x is the (operational) attenuation (dB);

B_x is the (operational) attenuation phase shift (rad).

EL FEXT = Equal Level Far-End Crosstalk

$= FEXT - \Gamma_{B1}$

ACR-F = Attenuation to Crosstalk Ratio in the Far-end (compare Signal to Crosstalk Ratio and *EL FEXT*)

$= FEXT - \Gamma_{B2}$

Γ_{B1} = (complex) operational attenuation of system (1)

Γ_{B2}= (complex) operational attenuation of system (2)

AACR-F = Alien (exogenous) Attenuation to Crosstalk Ratio in the Far-end

PS = Power-Sum.

A.7.4 Complex image backward echo attenuation[12]

Complex image backward echo attenuation is the quotient of the (unreflected) square root of the power wave sent into the input of the two-port and the square root of the power wave due to reflections received from the input expressed in dB and radians.

Compare with Clause A.6

A.7.5 Complex image forward echo attenuation[13]

Complex image forward echo attenuation is the quotient of the (unreflected) square root of the power wave (main signal) received from the output of a two-port and echo square root of the power waves of the main signal from multiple reflection points following the main signal expressed in dB and radians.

A.7.6 Backward and forward echo attenuation or loss of a transmission line

$\Gamma_{wr} = A_{wr} + jB_{wr}$ = Complex image backward echo attenuation or loss (structural return loss)

(Complex image backward echo attenuation[13])

12 Komplexe Wellen-Rückfluss-Dämpfung.

13 Komplexe Wellen-Mitfluss-Dämpfung.

$\Gamma_{wq} = A_{wq} + jB_{wq}$ = Complex image forward echo attenuation or loss

(Complex image forward echo[14])

The relation between structural return loss A_{wp} and forward echo loss A_{wq} of regular and periodic reflections of a transmission line is

$$A_{wq} \approx 2\,A_{wr} - 20\log_{10}(\,2\alpha L - 1\,)\ [\text{dB}]$$

When 2 × (attenuation of the transmission line) = $2\alpha L >> 1$

$2\alpha L$ in neper [Np]. $1\ \text{dB}$ = 0,115 Np

See [6][14].

14 References in square brackets refer to the Bibliography.

Annex B
(normative)

Image transmission parameters/quantities of a two-port and transmission line approximations

B.1 General

The image transmission parameters/quantities of a two-port are defined for the condition of no reflections at the input and output. This condition is achieved by terminating the input and output with their image or characteristic impedance.

B.2 Image transfer function

The image transfer function and associated terms are given below:

a) Image transfer function $T = \frac{\sqrt{P_{OUT}}}{\sqrt{P_{IN}}} = \frac{U_{OUT}}{U_{IN}} \cdot \frac{\sqrt{Z_{01}}}{\sqrt{Z_{02}}}$

b) Image transfer attenuation or loss $A = 20 \times \log_{10} \left| \frac{1}{T} \right|$

Z_{01} and Z_{02} are the image or characteristic impedances of the input or output of the two-port, equal to the input and output impedances when the opposite port is terminated with its image impedance.

$\sqrt{P_{IN}} = U_{IN} / \sqrt{Z_{01}}$ and $\sqrt{P_{OUT}} = U_{OUT} / \sqrt{Z_{02}}$ are the square roots of the complex input and output powers.

B.3 Image quantities

The transmission quantities of a two-port corresponding to the secondary parameters of a transmission line are shown in Table B.1. The corresponding high and low frequency approximations for the secondary parameters of a homogenous transmission line are also given.

When $R/\omega L$ and $G/\omega C$ are smaller than 0,1 at high frequencies, the deviations are smaller than 1 %. The same is valid for low frequencies when $\omega L/R$ instead of $R/\omega L$ is smaller than 0,1.

R, L, G and C are the primary parameters – *resistance, inductance, conductance* and *capacitance* – per unit length of a homogenous transmission line. The secondary parameters are then also per unit length

See [6].

Table B.1 – Transmission quantities of a two-port and homogeneous transmission line

Transmission quantity	Two-port	Transmission lines at high frequencies $R/\omega L$ and $G/\omega C < 0,1$	Transmission lines at low frequencies $\omega L/R$ and $G/\omega C < 0,1$
Complex image attenuation *(Komplexe Wellen-Dämpfung)*	$\Gamma = 20 \times \log_{10} \left\| \frac{1}{T} \right\| [\text{dB}] + \arg \frac{1}{T} [\text{rad}] = A + \text{j} B$	$\gamma = \alpha + \text{j}\beta$	$\gamma = \alpha + \text{j}\beta$ $\alpha \approx \beta \approx \sqrt{\frac{\omega R C}{2}}$
Image attenuation [a]	$A = 20 \times \log_{10} \left\| \frac{1}{T} \right\| [\text{dB}]$	$\alpha = \frac{R/2}{\sqrt{L/C}} + G\sqrt{\frac{L}{C}}$	$\alpha \approx \sqrt{\frac{\omega R C}{2}}$
Image phase shift [b]	$B = \arg \frac{1}{T} [\text{rad}]$	$\beta \approx \sqrt{L_\infty C} \left(1 + \frac{R}{2\omega L}\right)$	$\beta \approx \sqrt{\frac{\omega R C}{2}}$
Image phase propagation time or delay	$\tau_\text{p} = \frac{B}{\omega}$	$\tau_\text{p} \approx \sqrt{L_\infty C} \left(1 + \frac{R}{2\omega L}\right)$	$\tau_\text{p} \approx \sqrt{\frac{RC}{2\omega}}$
Image group propagation time or delay	$\tau_\text{s} = \frac{\text{d}B}{\text{d}\omega}$	$\tau_\text{g} \approx \sqrt{L_\infty C} \left(1 + \frac{R}{4\omega L}\right)$	
Image phase velocity	$v_\text{p} = \frac{1}{\tau_\text{p}}$		
Image group velocity	$v_\text{g} = \frac{1}{\tau_\text{g}}$		
Complex characteristic impedance [c]	$Z_\text{o} = \frac{U_\text{i}}{I_\text{i}}$	$Z_\text{o} = \text{Re}\, Z_\text{o} + \text{j}\, \text{Im}\, Z_\text{o}$ $\approx \sqrt{\frac{L_\infty}{C}} \left(1 + \frac{R}{2\omega L}\right)$ $- \text{j} \sqrt{\frac{L_\infty}{C}} \left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)$	$Z_\text{o} = \text{Re}\, Z_\text{o} + \text{j}\, \text{Im}\, Z_\text{o}$ $\text{Re}\, Z_\text{o} \approx -\text{Im}\, Z_\text{o} \approx \sqrt{\frac{R}{2\omega C}}$
Complex operational attenuation [d]	$\Gamma_\text{B} = A_\text{B} + \text{j} B_\text{B}$		
Operational attenuation [e]	$A_\text{B} = 20 \times \log_{10} \left(\frac{V_\text{i1}}{V_\text{r2}} \Big\|_{V_\text{i2}=0} \right)$		
Operational phase shift [f]	$B_\text{B} = \arg \left(\frac{V_\text{i1}}{V_\text{r2}} \Big\|_{V_\text{i2}=0} \right)$		

a Wellen-Dämpfung
b Wellen-Phasen-Schiebung
c Komplexer Wellen-Widerstand
d Komplexe Betriebs-Dämpfung
e Betriebs-Dämpfung
f Betriebs- Phasen-Schiebung

Annex C
(normative)

Two-port theory and fundamental concepts in transmission engineering

C.1 General

Annex C has two main goals. It lays the foundation for the fundamental terms and definitions to be used world-wide in describing the transmission properties of a two-port or quadripole, and builds a bridge between the classical quadripole theory and the scattering matrix presentation, which is based on the incident and reflecting square root of power waves at the input and output of a two-port. Finally, it is shown that the two concepts are bound together by simple equations, which are fundamentally identical.

The two-port, or quadripole theory was originally developed for voice and carrier technologies, transmission and later for microwaves, but it can be used for the whole frequency range and for various applications.

In the following clauses, the term two-port will be used exclusively.

C.2 Termination and load of a two-port

A termination is a one-port device which is connected to the end of a two-port and has the same nominal impedance as the line in order to avoid major reflections.

A load is a one-port device which is connected to the end of a two-port where no assumptions have been made about the matching or reflections.

C.3 Transfer equations for a passive two-port

For a passive impedance-symmetrical two-port (see Clause C.5 and Figure C.1), the following equations are valid:

$$U_1 = U_i + U_r \tag{C.1}$$

$$I_1 = I_i - I_r = \frac{U_i}{Z_0} - \frac{U_r}{Z_0} \tag{C.2}$$

$$U_2 = U_i e^{-\Gamma} + U_r e^{\Gamma} \tag{C.3}$$

$$I_2 = I_1 e^{-\Gamma} - I_r e^{\Gamma} \tag{C.4}$$

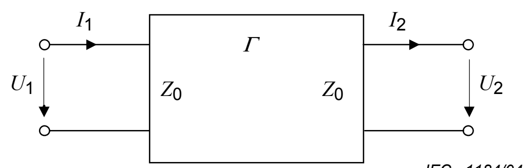


Figure C.1 – A two-port or quadripole

Where Z_0 is the image impedance of the two-port, $\Gamma = A + jB$ is the complex image attenuation or the image transfer constant. It equals the complex image attenuation of a two-port terminated in its image impedance (see Clause C.8). U_i and I_i represent the incident voltage and current waves fed to the input of the two-port, while U_r and I_r represent the voltage and current waves reflected back to the input from the output of the two-port. By solving Equations (C.1) and (C.2) for U_i, U_r, I_i and I_r and by substituting these into Equations (C.3) and (C.4), we obtain the actual voltage and current at the output terminals:

$$U_2 = U_1 \cosh\Gamma - Z_0 I_1 \sinh\Gamma \tag{C.5}$$

$$I_2 = I_1 \cosh\Gamma - \frac{U_1}{Z_0}\sinh\Gamma \tag{C.6}$$

By solving Equations (C.5) and (C.6) for U_1 and I_1 we obtain

$$U_1 = U_2 \cosh\Gamma + Z_0 I_2 \sinh\Gamma \tag{C.7}$$

$$I_1 = I_2 \cosh\Gamma + \frac{U_2}{Z_0}\sinh\Gamma \tag{C.8}$$

From which we can deduce that Equations (C.7) and (C.8) for input terminals can be obtained from Equations (C.5) and (C.6) for output terminals by interchanging the voltages, by interchanging the currents and by replacing Γ with $-\Gamma$.

From Equations (C.5), (C.6), (C.7) and (C.8), we can also solve the currents expressed by means of the voltages, as well as the voltages expressed by means of the currents:

$$I_1 = \frac{U_1}{Z_0}\coth\Gamma - \frac{U_2}{Z_0}\frac{1}{\sinh\Gamma} \tag{C.9}$$

$$I_2 = \frac{U_1}{Z_0}\frac{1}{\sinh\Gamma} - \frac{U_2}{Z_0}\coth\Gamma \tag{C.10}$$

$$U_1 = Z_0 I_1 \coth\Gamma - Z_0 I_2 \frac{1}{\sinh\Gamma} \tag{C.11}$$

$$U_2 = Z_0 I_1 \frac{1}{\sinh\Gamma} - Z_0 I_2 \coth\Gamma . \tag{C.12}$$

C.4 Chain matrix

Equations (C.7) and (C.8) are presented in matrix form in Equation (C.13):

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh\Gamma & Z_0 \sinh\Gamma \\ \frac{1}{Z_0}\sinh\Gamma & \cosh\Gamma \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.13}$$

Here, the multiplier matrix is called the chain matrix and is generally expressed as

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.14}$$

where, A, B, C and D, forming the chain matrix are called the transfer parameters. They are bound to each other by the relation:

$$AD - BC = 1 \tag{C.15}$$

The transfer parameters can be calculated by alternately considering the output of the two-pole either as short-circuited or open-circuited, whereby

$$A = \left(\frac{U_1}{U_2}\right)_{I_2=0} \qquad B = \left(\frac{U_1}{I_2}\right)_{U_2=0}$$
$$C = \left(\frac{I_1}{U_2}\right)_{I_2=0} \qquad D = \left(\frac{I_1}{I_2}\right)_{U_2=0} \tag{C.16}$$

The chain matrix is well suited for the examination of cascaded two-ports.

An impedance-unsymmetrical two-port (see Clause C.5) can be treated as a symmetrical one by cascading it (as shown by Figure C.2) with an ideal transformer with a turns ratio, K, of

$$K = \frac{N_1}{N_2} = \frac{1}{n} = \sqrt{\frac{Z_{01}}{Z_{02}}} \tag{C.17}$$

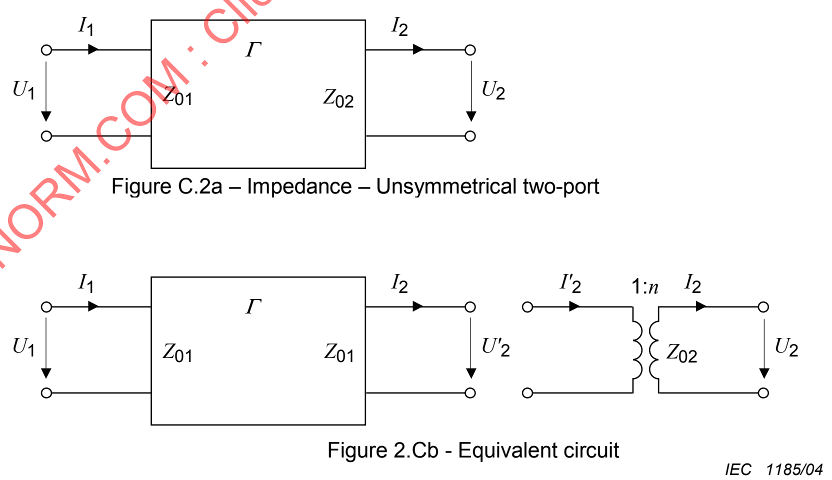


Figure C.2 – An impedance-unsymmetrical two-port (a) with its equivalent circuit (b)

We are here concerned with the cascading (or chaining) of two-ports, whereby the calculations can be appropriately carried out by means of chain matrices.

Let us suppose two, two-ports with the chain matrices A_1 and A_2 being interconnected as shown by Figure C.3:

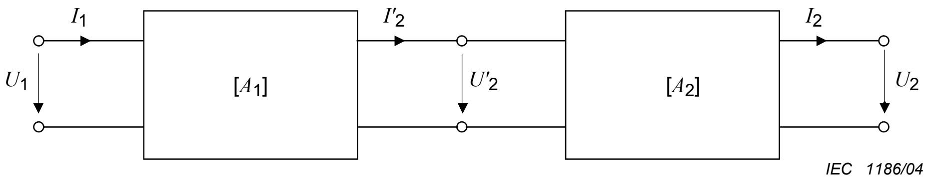


Figure C.3 – Two chained two-ports

The matrix equations, with the direction arrows as indicated in Figure C.3, are as follows:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = [A_1] \begin{bmatrix} U_2' \\ I_2' \end{bmatrix} \qquad \begin{bmatrix} U_2' \\ I_2' \end{bmatrix} = [A_2] \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.18}$$

The combining of Equations (C.18) yields:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = [A_1][A_2] \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = [A] \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.19}$$

where $[A] = [A_1][A_2]$.

The matrix A is hence obtained as a product between the chain matrices of the two-ports to be chained.

The turns ratio, K, of the transformer in Figure C.2 can be rewritten as

$$K = \frac{1}{n} = \frac{U_2'}{U_2} = \frac{I_2}{I_2'} = \sqrt{\frac{Z_{01}}{Z_{02}}} \tag{C.20}$$

and the transfer equation of the transformer is obtained in the matrix form:

$$\begin{bmatrix} U_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{Z_{01}}{Z_{02}}} & 0 \\ 0 & \sqrt{\frac{Z_{02}}{Z_{01}}} \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = [A_2] \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \tag{C.21}$$

In accordance with Equation (C.13), the chain matrix A_1 of a symmetrical two-port is equal to

$$[A_1] = \begin{bmatrix} \cosh \Gamma & Z_{01} \sinh \Gamma \\ \frac{1}{Z_{01}} \sinh \Gamma & \cosh \Gamma \end{bmatrix} \tag{C.22}$$

The matrix $[A]$ thus becomes:

$$[A]=[A_1][A_2]=\begin{bmatrix}\cosh\Gamma & Z_{01}\sinh\Gamma\\ \frac{1}{Z_{01}}\sinh\Gamma & \cosh\Gamma\end{bmatrix}\begin{bmatrix}\sqrt{\frac{Z_{01}}{Z_{02}}} & 0\\ 0 & \sqrt{\frac{Z_{02}}{Z_{01}}}\end{bmatrix}=\begin{bmatrix}\sqrt{\frac{Z_{01}}{Z_{02}}}\cosh\Gamma & Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\sinh\Gamma\\ \frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\sinh\Gamma & \sqrt{\frac{Z_{02}}{Z_{01}}}\cosh\Gamma\end{bmatrix} \quad \text{(C.23)}$$

The transfer equations of an impedance-unsymmetrical two-port can be written in the matrix form as follows:

$$\begin{bmatrix}U_1\\ I_1\end{bmatrix}=\begin{bmatrix}\sqrt{\frac{Z_{01}}{Z_{02}}}\cosh\Gamma & Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\sinh\Gamma\\ \frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\sinh\Gamma & \sqrt{\frac{Z_{02}}{Z_{01}}}\cosh\Gamma\end{bmatrix}\begin{bmatrix}U_2\\ I_2\end{bmatrix} \quad \text{(C.24)}$$

This matrix equation can also be solved for U_2 and I_2.

$$\begin{bmatrix}U_2\\ I_2\end{bmatrix}=[A]^{-1}\begin{bmatrix}U_1\\ I_1\end{bmatrix} \quad \text{(C.25)}$$

$$\begin{bmatrix}U_2\\ I_2\end{bmatrix}=\begin{bmatrix}\sqrt{\frac{Z_{02}}{Z_{01}}}\cosh\Gamma & -Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\sinh\Gamma\\ -\frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\sinh\Gamma & \sqrt{\frac{Z_{01}}{Z_{02}}}\cosh\Gamma\end{bmatrix}\begin{bmatrix}U_1\\ I_1\end{bmatrix} \quad \text{(C.26)}$$

From the matrix Equations (C.24) and (C.26), we can obtain the following transfer equations for an impedance-unsymmetrical two-port:

$$U_1=\sqrt{\frac{Z_{01}}{Z_{02}}}\,U_2\cosh\Gamma+Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\,I_2\sinh\Gamma \quad \text{(C.27)}$$

$$I_1=\sqrt{\frac{Z_{02}}{Z_{01}}}\,I_2\cosh\Gamma+\frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\,U_2\sinh\Gamma \quad \text{(C.28)}$$

$$U_2=\sqrt{\frac{Z_{02}}{Z_{01}}}\,U_1\cosh\Gamma-Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}}\,I_1\sinh\Gamma \quad \text{(C.29)}$$

$$I_2=\sqrt{\frac{Z_{01}}{Z_{02}}}\,I_1\cosh\Gamma-\frac{1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\,U_1\sinh\Gamma \quad \text{(C.30)}$$

The end results obtained can also be obtained direct from the transfer equations of an impedance-symmetrical two-port on the basis of Figure C.2.

By solving Equations (C.27), (C.28), (C.29) and (C.30), currents can be expressed by means of voltages or vice-versa, resulting in the following expressions:

$$I_1=\frac{U_1}{Z_{01}}\coth\Gamma-\frac{U_2}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\,\frac{1}{\sinh\Gamma} \quad \text{(C.31)}$$

$$I_2 = \frac{U_1}{Z_{01}}\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{1}{\sinh \Gamma} - \frac{U_2}{Z_{02}}\coth \Gamma \tag{C.32}$$

$$U_1 = Z_{01} I_1 \coth \Gamma - Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}} I_2 \frac{1}{\sinh \Gamma} \tag{C.33}$$

$$U_2 = Z_{01}\sqrt{\frac{Z_{02}}{Z_{01}}} I_1 \frac{1}{\sinh \Gamma} - Z_{02} I_2 \coth \Gamma \tag{C.34}$$

NOTE A short reminder on matrices:

$$M_2 = K * M_1$$
$$M_1 = K^{-1} * M_2$$

When $M_2 = K * M_1$, where M_1, M_2 and K are matrices,

then $M_1 = K^{-1} * M_2$, where K^{-1} is the inverse matrix of K.

When

$$[K] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

the inverse is

$$\left[K^{-1}\right] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{\Delta} \times \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

where the determinant is $\Delta = AD - BC$.

C.5 The symmetries and impedances of a two-port

Let us examine the two, two-ports illustrated in Figures C.4 and C.5:

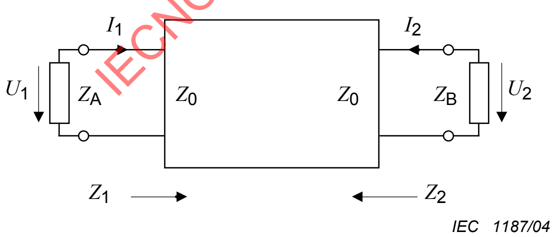


Figure C.4 – An impedance-symmetrical two-port with $Z_1 = Z_2$, when $Z_A = Z_B$

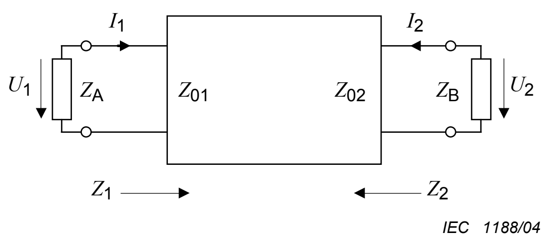


Figure C.5 – An impedance-unsymmetrical two-port for which $Z_1 \neq Z_2$ when $Z_A = Z_B$

The two-port in accordance with Figure C.4 is referred to as impedance-symmetrical or port-symmetrical, while the two-port of Figure C.5 is called impedance-unsymmetrical or port-unsymmetrical.

If the complex operational loss (see Clause A.2) in the direction A→B is equal to that in the direction B→A for any values of generator and terminating impedance, then the two-port is referred to as transfer-symmetrical or reciprocal. Two-ports that consist of passive components (except gyrators) are always reciprocal. A two-port with none of its properties depending on the direction of transmission is both reciprocal and impedance-symmetrical. Such a two-port is referred to as longitudinally symmetrical. The input terminals of a two-port are earth-symmetrical, if the admittances measured at each input terminal relative to earth are equal. In this case we speak of transversal symmetry of the two-port [3].

In addition to the complex image attenuation $\Gamma = A + jB$, there is another characteristic quantity for a two-port, that is, the image impedance. The image impedances Z_{01} and Z_{02} of a two-port in accordance with Figure C.5 can be determined by means of short-circuit and open-circuit measurements:

$$Z_{01} = \sqrt{Z_{1s} Z_{1o}} \qquad Z_{02} = \sqrt{Z_{2s} Z_{2o}}$$

where the subscripts s and o refer to the short-circuit and open-circuit conditions, respectively.

Let us recall the Equations (C.7) and (C.8) valid for a longitudinally symmetrical two-port:

$$\begin{aligned} U_1 &= U_2 \cosh \Gamma + Z_0 I_2 \sinh \Gamma \\ I_1 &= I_2 \cosh \Gamma + \frac{U_2}{Z_0} \sinh \Gamma \end{aligned} \qquad \text{(C.35)}$$

Taking into account that $U_2 = Z_B I_2$, (see Figure C.6), we obtain from Equations (C.35) the input impedance of the two-port:

$$Z_1 = \frac{U_1}{I_1} = Z_0 \frac{Z_B + Z_0 \tanh \Gamma}{Z_0 + Z_B \tanh \Gamma} \qquad \text{(C.36)}$$

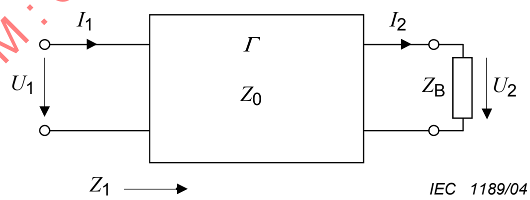


Figure C.6 – A two-port terminated with an impedance Z_B

Hence, the input impedance Z_1 depends on the properties of the two-port as well as on the terminating load impedance Z_B. It can be shown that when the attenuation A is high, Z_1 is only slightly affected by Z_B. From Equation (C.36), we see that $Z_1 \approx Z_0$, when tanh $|\Gamma| \approx 1$, i.e. when A >2 Np. The input impedance is then solely determined by the properties of the two-port. A two-port is called electrically short, when A << 2 Np and B << π/2, and correspondingly electrically long, when $A \geq$ 2 Np and $B \geq \pi/2$.

For two ports with small losses, i.e. $A \to 0$, tanh $\Gamma \to$ j tan B. If we than replace $B = 2\pi l/\lambda$ we can obtain, that for even multiples of the half wavelength $l = n\ \lambda/2$ tan B = 0 and thus

$$Z_1 = Z_B \quad \text{for} \quad l = n \lambda/2 \qquad \text{(C.37)}$$

I.e. any load impedance at the output of the two-port appears unchanged at the input of the two-port. On the other hand, for odd multiples of the quarter wavelength $l = (2n+1)\ \lambda/4$ $\tan B \rightarrow \infty$ and thus:

$$Z_1 = \frac{Z_0^2}{Z_B} \quad \text{for} \quad l = (2n+1)\lambda/4 \tag{C.38}$$

i.e. the impedance Z_B can be transformed into an impedance Z_1. This is only feasible at the exact frequency for which the length of the lossless line is $\lambda/4$, corresponding to a so-called quarter-wavelength transformer.

When the output is short-circuited ($Z_B = 0$), we have

$$Z_{1s} = Z_0 \tanh \Gamma \tag{C.39}$$

and when the output is open ($Z_B = \infty$), we have

$$Z_{1o} = Z_0 \frac{1}{\tanh \Gamma} \tag{C.40}$$

Equations (C.39) and (C-40) reveal that also the Z_0 and Γ of a longitudinally symmetrical two-port can be determined from the short-circuit and open-circuit impedances.

C.6 Impedance matching

If the image impedances of the two-ports to be cascaded differ from each other, reflections will be generated at the interconnection points, and those reflections then affect the uniformity of transmission. In telecommunication engineering, to avoid reflections in transmission, it is important that the impedances of the consecutive sections included in a transmission path are carefully matched to each other, i.e. the characteristic impedances of the devices to be cascaded shall very closely equal each other. A non-distorted transmission will only be possible under such conditions. However, it should be noted that one single major mismatch can be allowed within each repeater section; for example, provided that all other mismatches are small enough, because at least two mismatches are required for the generation of a propagating, signal-distorting forward-echo.

By substituting the quantities $U_2 = I_2\, Z_0$, which correspond to a proper matching ($Z_B = Z_0$) into Equations (C.35), we obtain:

$$U_1 = U_2\, e^{\Gamma} = I_2 Z_0\, e^{\Gamma} \tag{C.41}$$

$$I_1 = I_2\, e^{\Gamma} \tag{C.42}$$

from which it follows that the input impedance is

$$Z_1 = \frac{U_1}{I_1} = \frac{I_2\, Z_0\, e^{\Gamma}}{I_2\, e^{\Gamma}} = Z_0 \tag{C.43}$$

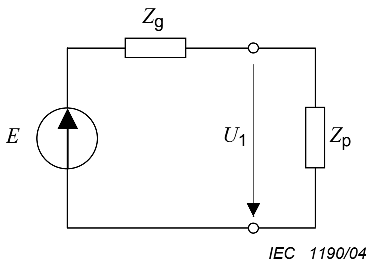
Hence the input impedance is under these conditions independent of Γ.

Correct matching enables the greatest possible complex power to be transmitted from a generator to the load. (In the literature, the term 'complex power' often refers to the quantity

UI^*, while the quantity UI is called the 'apparent power'. In transmission engineering, it is logical to use the term 'complex power' to denote the product of voltage and current phasors.)

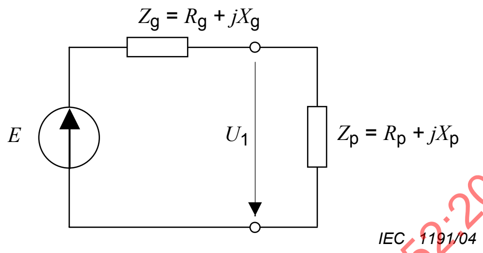
Hence,

$$P = UI \qquad \text{(C.44)}$$



$Z_g = Z_p$

Figure C.7 – Reflection loss matching



$Z_g = Z_p^*$ or $R_g = R_p$ with $X_g = X_p = 0$

Figure C.8 – Power matching for maximizing the effective power

The complex power (see Figures C.7 and C.8) obtained with the load Z_p is

$$P = \frac{E^2 Z_p}{(Z_g + Z_p)^2} \qquad \text{(C.45)}$$

which reaches a maximum when $Z_P = Z_g$, which yields

$$P_{max} = \frac{E^2}{4Z_g} \qquad \text{(C.46)}$$

With $Z_g = R_g + jX_g$ and $Z_p = R_p + jX_p$ the greatest possible effective power is absorbed by the load when $R_g = R_p$ and $j(X_g + X_p) = 0$. The condition is met when both imaginary parts are zeros, or when the impedances are complex conjugates, i.e. $Z_g = Z^*_p$. This kind of matching is called power matching. It is commonly used when matching transmitters to antennas, but, being normally valid at a single frequency only (the tuning frequency), it has found no applications in broad-band transmission techniques. Even a two-port (its output or input, respectively) can be considered as a power source or a load.

The input or output impedance of a two-port can be built out in such a way as to be resistive while being independent of frequency, under the condition that it is represented by a series combination of R and L, or R and C. For example, if an impedance $R + 1/j\omega C$ is connected in parallel with an impedance $R + j\omega L$ by choosing $C = L/R^2$, a frequency-independent resistive impedance R will be obtained.

C.7 Level concepts

The term 'level' is used to indicate a relative or an absolute value. If the power, voltage or current along a transmission system is concerned, one speaks of power, voltage or current levels.

When comparing the power, voltage or current at a measuring point with the respective quantity at the feeding point of the transmission system, we are concerned with a relative level, whereas, when the comparison is made to a standardized reference value, an absolute level will be obtained.

Levels are commonly expressed in decibels (dB), more seldom in nepers (Np). The use of nepers is actually restricted to some theoretical calculations. The units are related by

$$1 \text{ dB} = 0{,}05/\log_{10} \text{e} = 0{,}1151 \text{ Np or } 1 \text{ Np} = 20 \log_{10} \text{e} = 8{,}686 \text{ dB}$$

If P_x and V_x denote the power and the voltage at the measuring point, while P_A and V_A are the corresponding values at the feeding point (input) of the system, the relative power level is

$$N = 10 \times \log_{10} \frac{P_X}{P_A} \text{ [dB]} = \frac{1}{2} \times \ln \frac{P_X}{P_A} \text{ [Np]} \qquad \text{(C.47)}$$

and the relative voltage level is

$$N_v = 20 \times \log_{10} \frac{V_X}{V_A} \text{ [dB]} = \ln \frac{V_X}{V_A} \text{ [Np]} \qquad \text{(C.48)}$$

The relative level at the input of the system is always zero.

If P_1 and V_1 are the standardized reference values, the absolute power level is given by:

$$N = 10 \times \log_{10} \frac{P_X}{P_1} \text{ [dB]} = \frac{1}{2} \times \ln \frac{P_X}{P_1} \text{ [Np]} \qquad \text{(C.49)}$$

while the absolute voltage level is

$$N_v = 20 \times \log_{10} \frac{V_X}{V_1} \text{ [dB]} = \ln \frac{V_X}{V_{1A}} \text{ [Np]} \qquad \text{(C.50)}$$

In telecommunication engineering, the reference for absolute power levels is 1 mW and the reference for absolute voltage levels is 0,775 V, which corresponds to 1 mW in a 600 Ω load. Nowadays, voltage levels are seldom used in telecommunication engineering, to avoid confusion. There is a tendency towards an exclusive use of power levels.

In conjunction with broadcast relaying, community antennas and closed-link television systems, instead, voltage levels based on a reference of 1 µV have been adopted. A reference impedance of 75 Ω is implied, however only insofar as it relates to the last two systems, so that here one is actually concerned with power levels.

To discriminate between the above absolute levels with references 1 mW and 1 µV, respectively, the designations dBm (dB(mW)) and dB(µV) have been adopted. In transmission engineering, it proved advantageous to use a nominal level, reached when a power 1 mW is fed to the input or prevails at a fictive reference point in the system. Relative to this 1 mW point, the nominal level of the system is always 0 dB(mW) and is denoted 0 dBm0. In other words, the nominal level along the entire system can be thought to be 0 dBm0 (see Figure C.9). Hence designation –50 dBm0, for example, means a level which lies 50 dB below the nominal level of the system.

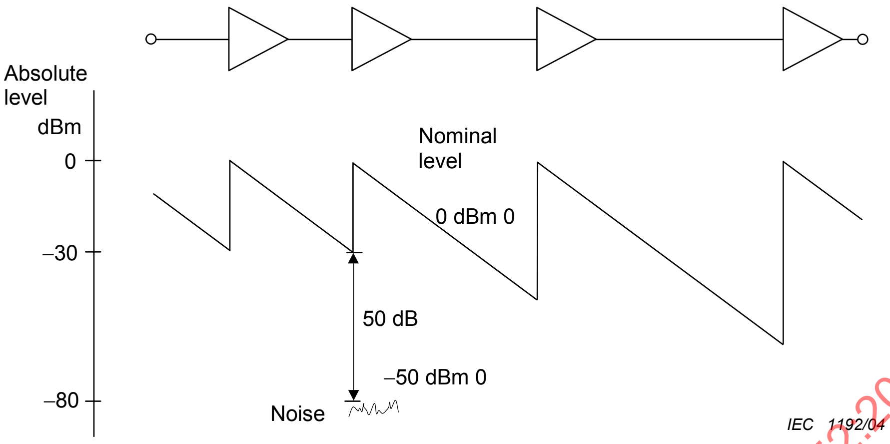


Figure C.9 – Absolute and nominal level in a system

In speech transmission, it is often appropriate to weight a disturbing noise signal in accordance with the sensitivity curve of the ear. Such a psophometrically weighted noise level, being, for example, 50 dB below the nominal level, is designated as –50 dB0p. The matter can also be expressed so that we have here a psophometrically weighted noise power reduced to the 0 dB(mW) point (1 mW point) and having a level 50 dB below 1 mW.

When it is necessary to emphasize that a level is a relative level or, respectively, a voltage level, designations dBr and dBu are employed.

C.8 Attenuation and gain concepts

The complex image attenuation or image transfer constant Γ of a two-port is defined as a logarithmic ratio between the power $P_1 = U_1 I_1$ fed to the input terminals and the power $P_2 = U_2 I_2$ obtained at the output, when the two-port is terminated in an impedance which is equal to the output image impedance of the two-port (see Figure C.10).

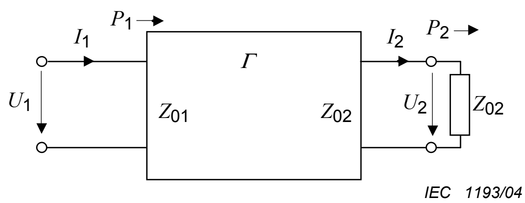


Figure C.10 – Definition of the complex image attenuation Γ of a two-port

$$\Gamma = A + \mathrm{j}B = 10 \times \log_{10} \frac{P_1}{P_2}\ [\mathrm{dB}] = \frac{1}{2} \times \ln \frac{P_1}{P_2}\ [\mathrm{Np}]$$

$$= 10 \times \log_{10} \frac{U_1 I_1}{U_2 I_2}\ [\mathrm{dB}] = \frac{1}{2} \times \ln \frac{U_1 I_1}{U_2 I_2}\ [\mathrm{Np}] \qquad (C.51)$$

$$\begin{aligned} \Gamma &= 20 \times \log_{10}\left(\frac{U_1}{U_2}\sqrt{\frac{Z_{02}}{Z_{01}}}\right)[\mathrm{dB}] = \ln\left(\frac{U_1}{U_2}\sqrt{\frac{Z_{02}}{Z_{01}}}\right)[\mathrm{Np}] \\ &= 20 \times \log_{10}\left(\frac{I_1}{I_2}\sqrt{\frac{Z_{01}}{Z_{02}}}\right)[\mathrm{dB}] = \ln\left(\frac{I_1}{I_2}\sqrt{\frac{Z_{01}}{Z_{02}}}\right)[\mathrm{Np}] \end{aligned} \tag{C.52}$$

Hence we have

$$A = 10 \times \log_{10}\left|\frac{P_1}{P_2}\right| [\mathrm{dB}] = \frac{1}{2} \times \ln\left|\frac{P_1}{P_2}\right| [\mathrm{Np}] \tag{C.53}$$

$$B = \frac{1}{2} \times arg\frac{P_1}{P_2} = \frac{1}{2} \times (\angle P_1 - \angle P_2) \tag{C.54}$$

A is the image attenuation and B is the image phase constant. If the two-port is impedance-symmetrical ($Z_{01} = Z_{02}$), the equations are more simple and we obtain the expression:

$$\begin{aligned} \Gamma &= 20 \times \log_{10}\frac{U_1}{U_2} [\mathrm{dB}] = \ln\frac{U_1}{U_2} [\mathrm{Np}] \\ &= 20 \times \log_{10}\frac{I_1}{I_2} [\mathrm{dB}] = \ln\frac{I_1}{I_2} [\mathrm{Np}] \end{aligned} \tag{C.55}$$

During actual operation, a two-port often lies between terminating devices with impedances differing from the image impedances of the two-port. It is then appropriate to speak of operational attenuation. The complex operational attenuation or complex operational transfer constant Γ_B is defined as a logarithmic ratio between the power $P_1 = E^2/(4 \times Z_g)$ fed by the generator to a load equal to its internal impedance Z_g, and the power $P_2 = U_2 I_2 = U_2^2/Z_p$ obtained to the load Z_p at the output of the two-port (see Figure C.11).

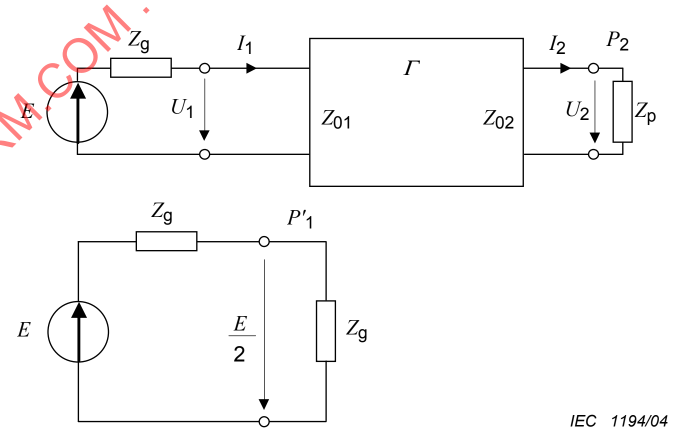


Figure C.11 – Definition of the complex operational attenuation of a two-port

$$\begin{aligned} \Gamma_B = A_B + jB_B &= 10 \times \log_{10}\frac{P_1'}{P_2}[\mathrm{dB}] = \frac{1}{2} \times \ln\frac{P_1'}{P_2}[\mathrm{Np}] \\ &= 20 \times \log_{10}\left(\frac{E}{2 \times U_2}\sqrt{\frac{Z_p}{Z_g}}\right)[\mathrm{dB}] = \ln\left(\frac{E}{2 \times U_2}\sqrt{\frac{Z_p}{Z_g}}\right)[\mathrm{Np}] \end{aligned} \tag{C.56}$$

$$A_B = 10 \times \log_{10} \left| \frac{P_1'}{P_2} \right| \text{ [dB]} = \frac{1}{2} \times \ln \left| \frac{P_1'}{P_2} \right| \text{[Np]} \quad \text{(C.57)}$$

$$B_B = \frac{1}{2} \times \arg \frac{P_1'}{P_2} = \frac{1}{2} (\angle P_1' - \angle P_2) \quad \text{(C.58)}$$

The expression $\frac{2 \times U_2}{E} \sqrt{\frac{Z_g}{Z_p}}$ is called the operational transfer constant H_B.

A_B is the operational attenuation and B_B the operational phase constant. If the impedances of the generator and the load are equal ($Z_p = Z_g$), then the equations are simplified and we obtain for the operational transfer constant:

$$\Gamma_B = 20 \times \log_{10} \frac{E}{2 \times U_2} \text{[dB]} = \ln \frac{E}{2 \times U_2} \text{[Np]} \quad \text{(C.59)}$$

The complex operational gain $-\Gamma_B$ is the negative of the complex operational attenuation Γ_B:

$$\begin{aligned} -\Gamma_B &= -A_B - jB_B = -10 \times \log_{10} \frac{P_1'}{P_2} \text{[dB]} = 10 \times \log_{10} \frac{P_2}{P_1'} \text{[dB]} \\ &= 20 \times \log_{10} \frac{2 \times U_2}{E} \sqrt{\frac{Z_g}{Z_p}} \text{[dB]} = \ln \frac{2 \times U_2}{E} \sqrt{\frac{Z_g}{Z_p}} \text{[Np]} \end{aligned} \quad \text{(C.60)}$$

$-A_B$ is the operational gain and $-B_B$ is the operational gain phase angle.

Residual attenuation: The amplifiers (repeaters) connected in a transmission line cancel a part of the attenuation caused by the lines. The remaining part is called the residual attenuation. The residual attenuation A is equal to the difference between the total attenuation of all lines and components in the transmission path, A_k and the sum of the gain of all amplifiers S_k in the transmission line:

$$A = A_k - S_k \quad \text{[dB]} \quad \text{(C.61)}$$

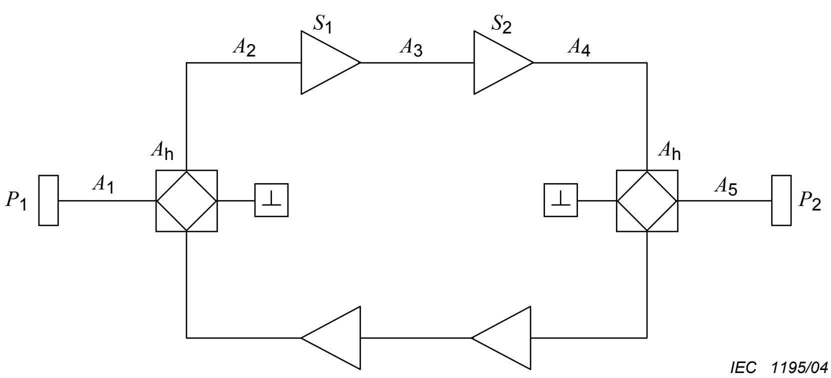


Figure C.12 – Definition of residual attenuation

The residual attenuation of the 2/4-wire line shown by Figure C.12 is equal to

$$A = (A_1 + A_2 + A_3 + A_4 + A_5 + 2 \times A_h) - (S_1 + S_2)$$

where

$A_{1...5}$ are the attenuations of different line sections;

A_h is the attenuation of the hybrid networks;

S_1 and S_2 are the gain of the respective repeater.

The reference equivalent is a measure for the speech-transmitting capabilities of a telephone connection. It is defined as the attenuation that must be added to the attenuation of a reference system so that the loudness of the speech through the damped reference system is equal to that through the actual system under examination. If the system under test is less sensitive than the reference system, the reference equivalent is considered to be positive.

As an international reference system, the NOSFER system (residing in the laboratories of CCITT) is employed. When measuring the sending reference equivalent (see Figure C.13), a person alternately speaks to the microphone of the system under test and to the microphone of NOSFER. There is a VU meter in the transmitting circuit of NOSFER that enables the speaker to exercise constant loudness. A listener adjusts the attenuator (A_R) in NOSFER in such a manner that equal loudness is experienced through both systems.

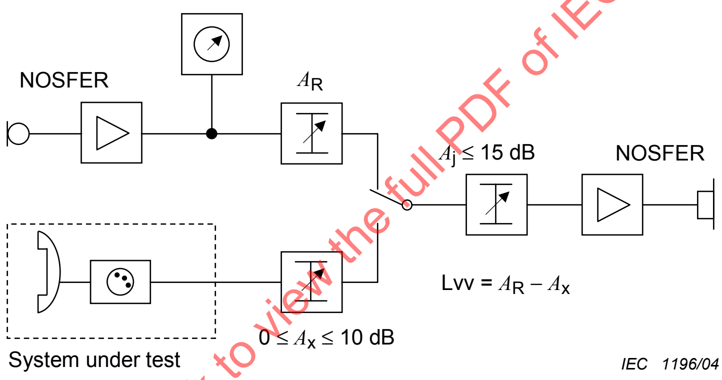


Figure C.13 – Measurement of the sending reference equivalent

When measuring the receiving reference equivalent (see Figure C.14), the speaker speaks into the NOSFER microphone, while the listener alternately listens through both systems, equalizing the loudness by means of an attenuator in similar fashion to the measurement of the sending reference equivalent.

The standard deviation of the test results in the case of a trained testing team is usually of the order (1,5 to 2,5) dB, while the 95 % margin lies within the range (±0,5 to ±4) dB.

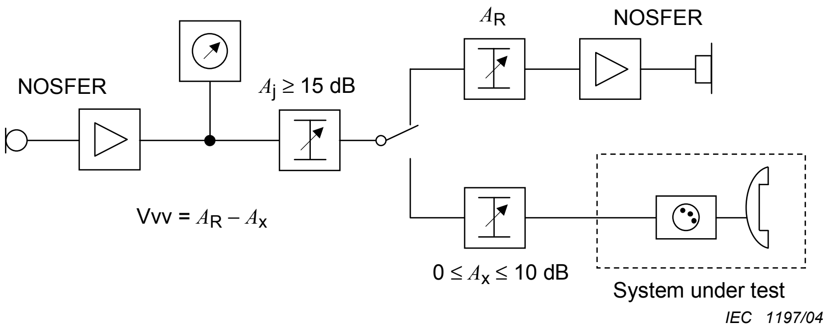


Figure C.14 – Measurement of the receiving reference equivalent

Attempts have been made to replace the subjective method of measurement by an objective one. One such method is at the moment under consideration in CCITT. However, the results obtained by objective methods have yet to coincide with adequate precision with those obtained by using the subjective method.

C.9 Concepts related to return loss and matching

Let us examine the circuit given in Figure C.15, where U_i denotes the incident voltage wave that reaches a reflection point, while U_r is the voltage wave reflected back from the point of reflection.

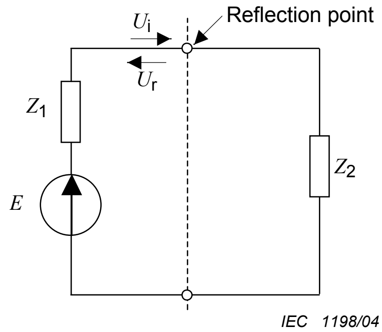


Figure C.15 – Definition of the complex return loss

The reflection coefficient ρ is the ratio between the reflected and incident waves:

$$\rho = \frac{U_r}{U_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{C.62}$$

The complex return loss Γ_r is correspondingly defined as

$$\begin{aligned} \Gamma_r &= A_r + j\,B_r = \ln\frac{1}{\rho} = \ln\left|\frac{1}{\rho}\right| [\text{Np}] + j\arg\frac{1}{\rho}\,[\text{rad}] \\ &= A_r + j\,B_r = 20\times\log_{10}\left|\frac{1}{\rho}\right| [\text{dB}] + j\arg\frac{1}{\rho}\,[\text{rad}] \end{aligned} \tag{C.63}$$

$$A_r = 20\times\log_{10}\left|\frac{Z_2 + Z_1}{Z_2 - Z_1}\right| [\text{dB}] = \ln\left|\frac{Z_2 + Z_1}{Z_2 - Z_1}\right| [\text{Np}] \tag{C.64}$$

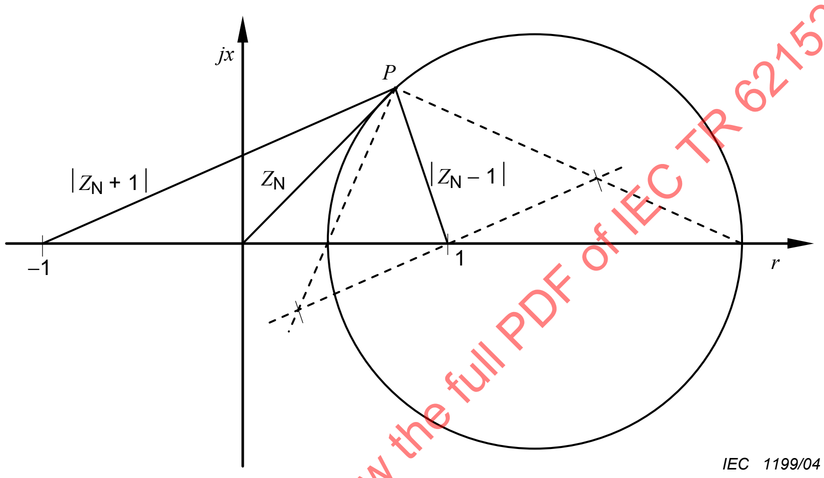
$$B_r = \arg\frac{Z_2 + Z_1}{Z_2 - Z_1} = \angle\frac{Z_2 + Z_1}{Z_2 - Z_1} [\text{rad}] \tag{C.65}$$

A_r is the return loss and B_r is the reflection phase constant.

Equation (C.64) for return loss can be rewritten in the form

$$A_r = 20\times\log_{10}\left|\frac{z_N + 1}{z_N - 1}\right| [\text{dB}] \tag{C.66}$$

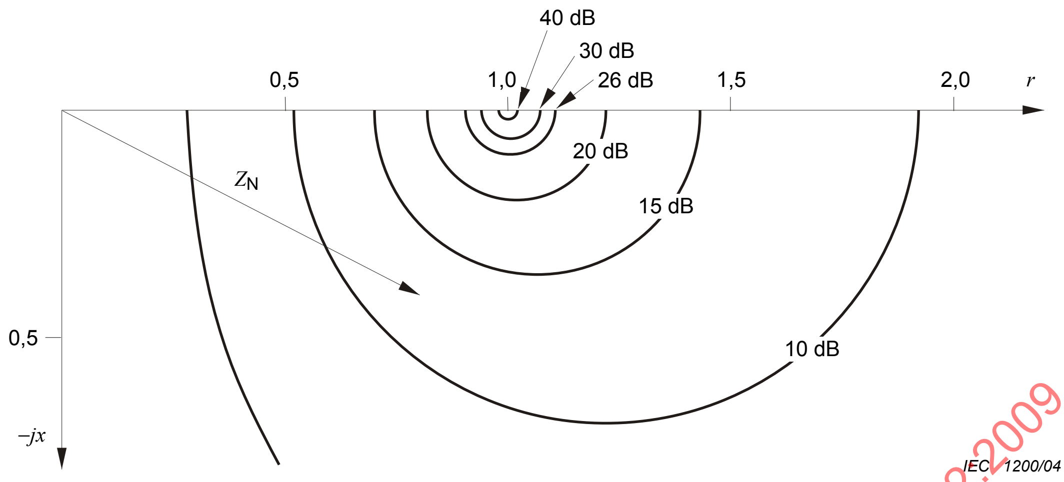
where $z_N = Z_2/Z_1$ is the normalized impedance. If the reflection coefficient is constant, then the term $|z_N + 1|/|z_N - 1|$ is also constant. In accordance with Figure C.16, the numerator and the denominator can be considered as sections of lines, which indicate the distances of the end point P of the vector z_N from the points (–1,0) and (1,0), respectively. All other points, for which the ratio between the distances from the points (–1,0) and (1,0) equals the above constant, are found by drawing through P a so-called Apollonius' circle. It is formed by the locus of points, for which the ratio of the distances from two fixed points is constant. The Apollonius' circle can be constructed by separating equal sections with length $|z_N - 1|$ on both sides of point (1,0) along a line, which is parallel to $|z_N + 1|$ and passes the point (1,0). When two lines are drawn through point P and the external ends of the above sections, two intersection points are obtained on the real axis. The distance between these intersection points determines the diameter of the Apollonius' circle.



The Apollonius circle is formed by the locus of points for which the ratio of distances from the points (–1,0) and (1,0) is constant.

Figure C.16 – Apollonius' circle

The points on the periphery of the circle in Figure C.16 represent a constant value of return loss. Inside the circle, the return loss is greater and, outside the circle, smaller than on the periphery. Because the circle is symmetrical in relation to the real axis, only one-half of the circle is usually drawn. By drawing several Apollonius' circles, each of them corresponding to a different value of return loss, a chart in accordance with Figure C.17 will be obtained. Any normalized impedance, z_N, drawn on the chart then directly gives the corresponding return loss in dB. In the example shown, $A_r \approx 12$ dB.



$A_r = 20 \times \log_{10} |z_N + 1| / |z_N - 1|$ where $z_N = r + jx$ is the normalized impedance

Figure C.17 – Return loss

When substituting $Z_2 = 0$ (short-circuit) or $Z_2 = \infty$ (open-circuit) into Equation (C.64), A_r will vanish. When $Z_2 = Z_1$ (proper matching), there will be no reflections and, consequently, $A_r = \infty$.

The complex reflection loss Γ_s is

$$\Gamma_s = A_s + \mathrm{j}\,B_s = \ln \frac{1}{\sqrt{1-\rho^2}} = \ln \left| \frac{1}{\sqrt{1-\rho^2}} \right| [\mathrm{Np}] + \mathrm{j} \arg \frac{1}{\sqrt{1-\rho^2}} [\mathrm{rad}]$$

$$= 20 \times \log_{10} \left| \frac{1}{\sqrt{1-\rho^2}} \right| [\mathrm{dB}] + \mathrm{j} \arg \frac{1}{\sqrt{1-\rho^2}} [\mathrm{rad}] \tag{C.67}$$

$$A_s = 20 \times \log_{10} \left| \frac{Z_2 + Z_1}{2 \times \sqrt{Z_2 Z_1}} \right| [\mathrm{dB}] = \ln \left| \frac{Z_2 + Z_1}{2 \times \sqrt{Z_2 Z_1}} \right| [\mathrm{Np}] \tag{C.68}$$

$$B_s = \arg \frac{Z_2 + Z_1}{2 \times \sqrt{Z_2 Z_1}} = \angle \frac{Z_2 + Z_1}{2 \times \sqrt{Z_2 Z_1}} [\mathrm{rad}] \tag{C.69}$$

A_s is the reflection loss and B_s is the reflection loss phase angle. The quantity Γ_s indicates how much the complex power (transferred through the reflection point to the actual load Z_2) has been attenuated in comparison with the unreflected complex power transmitted through the reflection point, if the load were equal to Z_1 (no reflection). Hence, Equation (C.68) indicates that, with proper matching, i.e. $Z_2 = Z_1$ and $A_s = 0$, there exists also a number of other impedance pairs for which the reflection loss is zero. This is shown in Figures C.18 and C.19, which is a combination of circles with constant return loss and curves for constant values of A_s, all in a complex plane.

The right-hand side of the complex plane can be transformed into a circle with unit radius and with centre at point (1,0), whereby we obtain a so-called Smith's chart (see Figure C.20) for transmission lines. There, the Apollonius' circles of constant return loss are transformed into concentric circles with central point (1,0), whereby the variation of impedance along the line, caused by the mismatch between the line and the load, can be directly read by following a circle that passes point P for normalized load impedance. One clock-wise turn corresponds to a half-wavelength toward the generator (see, for example, references [1] or [2]). If the line is lossy, the return loss does not remain constant when proceeding toward the generator, and the variation of impedance along the line then forms a converging spiral in the Smith chart.

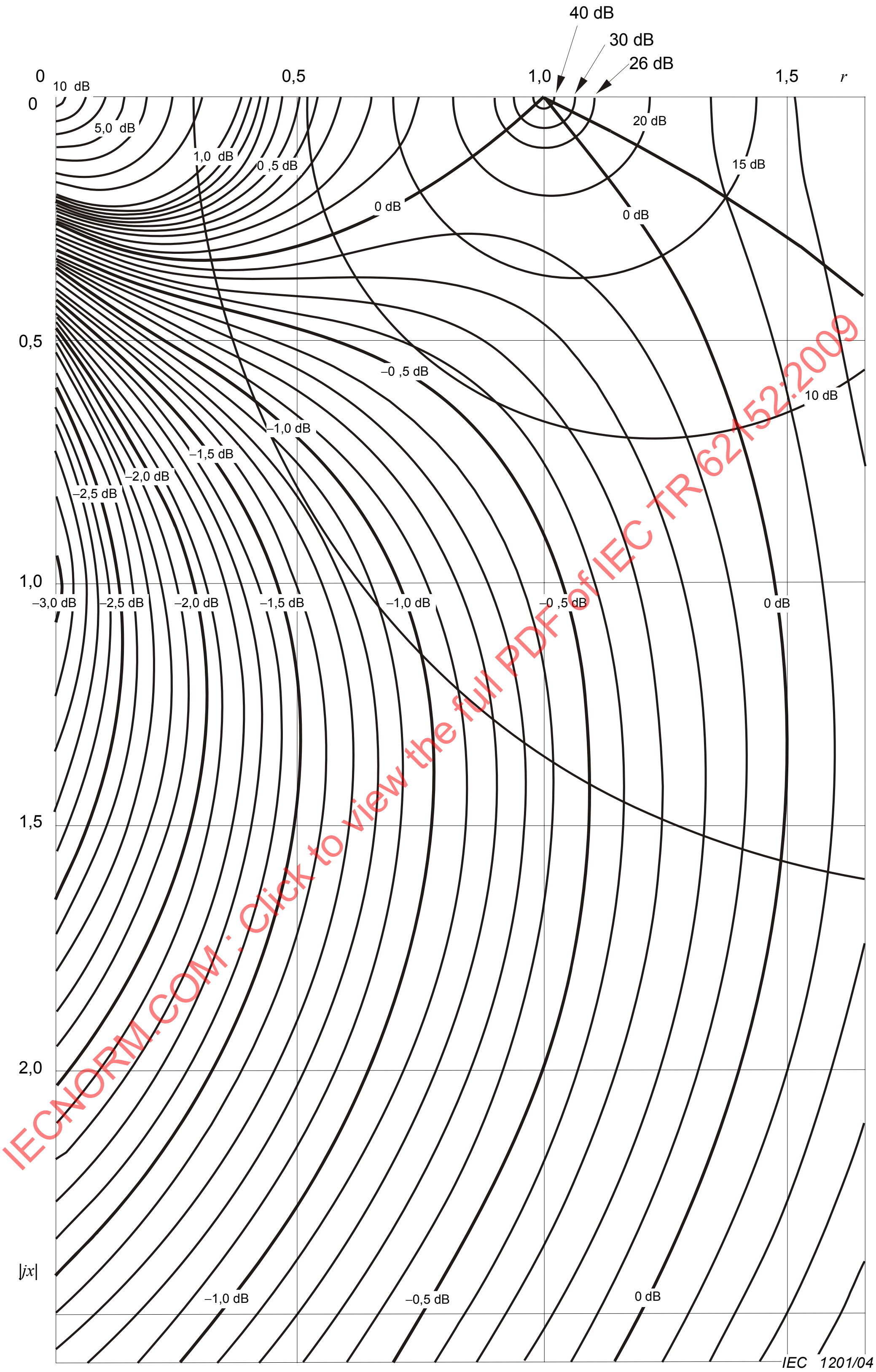
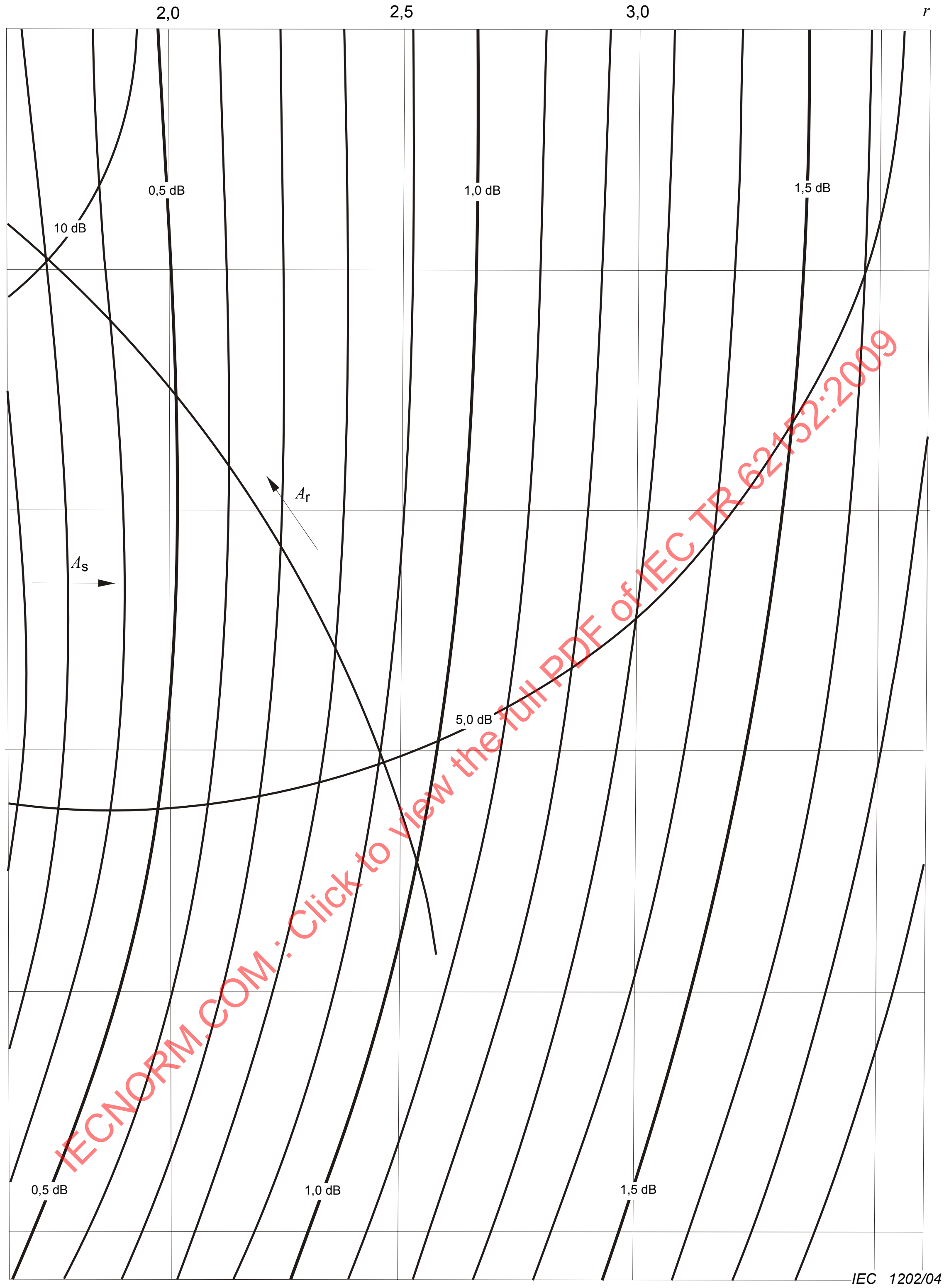


Figure C.18 – Curves for constant values of A_s or A_r in the complex plane



Reflection loss $A_s = 20 \times \log_{10} \left| \frac{z_N + 1}{2 \times \sqrt{z_N}} \right| [\text{dB}]$ Return loss $A_r = 20 \times \log_{10} \left| \frac{z_N + 1}{z_N - 1} \right| [\text{dB}]$

$$z_N = \frac{Z_2}{Z_1} \quad \text{(= normalized impedance)} = r + jx$$

Figure C.19 – Curves for constant values of A_s or A_r in the complex plane

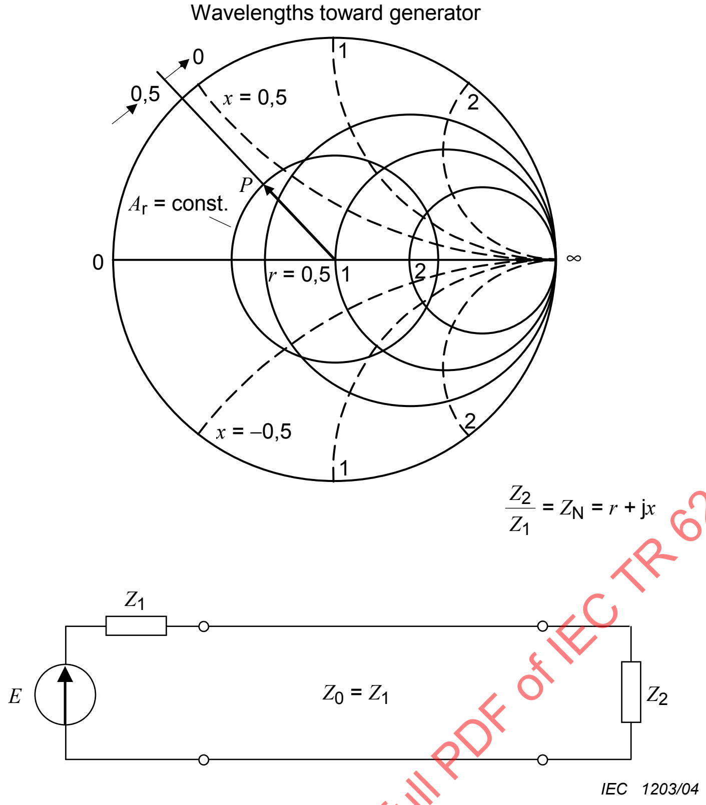


Figure C.20 – Smith chart for transmission lines

The voltage-standing-wave ratio $VSWR$ is the ratio between maximum and minimum values of the line voltages:

$$VSWR = \frac{U_{max}}{U_{min}} = \frac{1+|\rho|}{1-|\rho|} \qquad \text{(C.70)}$$

where the reflection coefficient $\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{z_N - 1}{z_N + 1}$

The absolute value of the reflection coefficient is calculated From Equation (C.69):

$$|\rho| = \frac{VSWR - 1}{VSWR + 1} \approx \frac{VSWR - 1}{2} \qquad \text{(C.71)}$$

when $VSWR \approx 1$ or $\rho << 1$.

C.10 Scattering parameters

C.10.1 Scattering parameters of a one-port

A one-port, as shown in Figure C.21, can be characterized by incident (i) and reflected (r) voltage, and current and square-root of power waves.

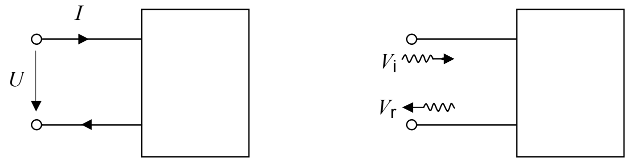


Figure C.21a –Voltage and current orientation **Figure C.21b – Incident and reflected square-root of power waves**

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Figure C.21 – A one-port

$$U = U_i + U_r \tag{C.72}$$

$$I = I_i - I_r = \frac{U_i}{R_0} - \frac{U_r}{R_0} \tag{C.73}$$

$$U_i = \frac{1}{2}(U + R_0 I) \tag{C.74}$$

$$U_r = \frac{1}{2}(U - R_0 I) \tag{C.75}$$

U and I are respectively the voltage and current at the terminals of the one-port and R_0 can be regarded as the image impedance of the one-port. Compare with the characteristic impedance of the homogenous transmission line shown in Figure C.22. For practical applications, it is advantageous to choose for the characteristic impedance nominal values, for example, 50 Ω, 75 Ω, 100 Ω, 120 Ω, 150 Ω.

This impedance is also used as reference impedance for measurements. This impedance does not necessarily correspond to the image impedance of the one-port, because V_i is defined as the unreflected square root of the power entering into the one-port and the square root of the fictive power, which is calculated or measured by matching the generator with this impedance.

Recalling that the square-root of power is:

$$\sqrt{P} = \sqrt{UI} = \frac{U}{\sqrt{R_0}} = \sqrt{R_0}\, I = V \tag{C.76}$$

$$\sqrt{P_i} = V_i = \frac{U_i}{\sqrt{R_0}} = \frac{1}{2} \times \left(\frac{U}{\sqrt{R_0}} + \sqrt{R_0}\, I\right) \tag{C.77}$$

$$\sqrt{P_r} = V_r = \frac{U_r}{\sqrt{R_0}} = \frac{1}{2} \times \left(\frac{U}{\sqrt{R_0}} - \sqrt{R_0}\, I\right) \tag{C.78}$$

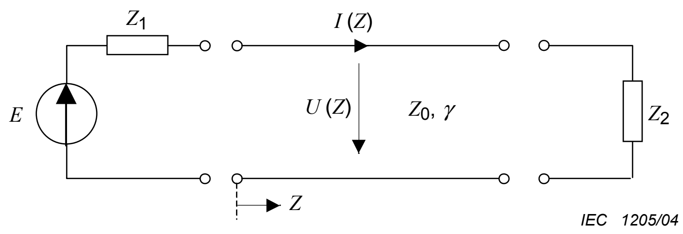

Figure C.22 – Homogenous transmission line

The relation between the incident and the reflected wave can be expressed by means of the scattering parameter S:

$$V_{\mathrm{r}} = S\,V_{\mathrm{i}} \tag{C.79}$$

The parameter S is here identical to the reflection coefficient ρ, which equally represents the ratio of the reflected voltage to the incident voltage at the reflection plane (see Clause C.9). From the definition for V_{i} and V_{r}, it follows that

$$\frac{U}{\sqrt{R_0}} - \sqrt{R_0}\,I = S\,(\frac{U}{\sqrt{R_0}} + \sqrt{R_0}\,I) \tag{C.80}$$

the solution of which gives

$$S = \frac{Z - R_0}{Z + R_0} \tag{C.81}$$

where $Z = U/I$ is the input impedance of the one-port.

If $Z = R_0$, the voltage of the reflected wave is V_{r} = 0. The inverse value of S, when expressed in dB or Np and radians, is called the complex return loss Γ_{r} (compare with Equation (C.63)):

$$\begin{aligned} \Gamma_{\mathrm{r}} &= 20 \times \log_{10}\left|\frac{1}{S}\right| [\mathrm{dB}] + \mathrm{j}\,\arg\left(\frac{1}{S}\right)[\mathrm{rad}] \\ &= \ln\left|\frac{1}{S}\right| [\mathrm{Np}] + \mathrm{j}\,\arg\left(\frac{1}{S}\right)[\mathrm{rad}] \end{aligned} \tag{C.82}$$

V_{i} at a one-port, which is fed from a generator with an internal impedance Z_{g} equal to the image impedance of the one-port Z_0, is:

$$V_{\mathrm{i}} = \frac{E}{2 \times \sqrt{Z_{\mathrm{g}}}} \tag{C.83}$$

By definition:

$$V_{\mathrm{i}} = \frac{1}{2} \times (\frac{U}{\sqrt{R_0}} + \sqrt{R_0}\,I) \tag{C.84}$$

Figure C.23 yields

$$U = E - I Z_g \tag{C.85}$$

and

$$I = \frac{E}{Z_g + Z} \tag{C.86}$$

Equation (C.83) is obtained when U and I are substituted into Equations (C.84), (C.85), (C.86) and (C.87).

$$V_i = \frac{1}{2\times\sqrt{R_0}}\left[\left(E - \frac{E\,Z_g}{Z_g + Z}\right) + \frac{R_0 E}{Z_g + Z}\right] = \frac{E}{2\times\sqrt{R_0}}\left(1 + \frac{R_0 - Z_g}{Z_g + Z}\right) = \left.\frac{E}{2\times\sqrt{Z_g}}\right|_{Z_g = R_0} \tag{C.87}$$

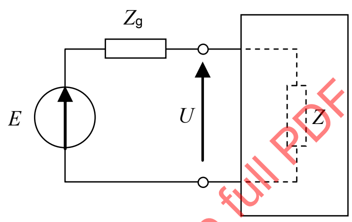

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Figure C.23 – One-port fed from a generator with source impedance Z_g

The reflected wave vanishes, if $Z = R_0$. On the other hand, there are no reflections between the generator and the load, if their impedances are equal, i.e. if $Z_g = Z$. This condition is equivalent to a properly matched generator and the load impedance.

The maximum effective power is transmitted to the load, when $Z_g = Z^*$ (see Clause C.6), whereas the maximum complex power is reached when $Z_g = Z$. In accordance with Equations (C.77) and (C.83), the maximum complex power is

$$P' = V_i^2 \tag{C.88}$$

From Equations (C.75) and (C.76) we obtain:

$$U = (V_i + V_r)\sqrt{R_0} \quad \text{and} \quad I = (V_i - V_r)/\sqrt{R_0} \tag{C.89 and C.90}$$

This yields the expression for the complex power absorbed by the one-port, which is represented by the load:

$$P = UI = V_i^2 - V_r^2 \tag{C.91}$$

Substitution of Equation (C.80) and (C.82) yields:

$$P = V_i^2(1 - S^2) = V_i^2\left[1 - \left(\frac{Z - Z_g}{Z + Z_g}\right)^2\right] \tag{C.92}$$

If the impedance Z_g of the generator that feeds the one-port is taken as reference impedance then the maximum complex power at the load is:

$$P' = V_i^2 \tag{C.93}$$

We obtain then the ratio between the complex power absorbed in the one-port and the actual complex power if the one-port is represented by the reference impedance Z:

$$\frac{P'}{P} = \frac{(Z + Z_g)^2}{4 \times Z Z_g} = \frac{1}{1 - S^2} \tag{C.94}$$

Compare to Equation (C.66). Expressed in logarithmic units, this is called the complex reflection loss:

$$\begin{aligned} \Gamma_s &= 20 \times \log_{10}\left|\frac{Z + Z_g}{2 \times \sqrt{Z Z_g}}\right| [\text{dB}] + \text{j}\arg\left(\frac{Z + Z_g}{2 \times \sqrt{Z Z_g}}\right) [\text{rad}] \\ &= -10 \times \log_{10}\left|1 - S^2\right| [\text{dB}] - \text{j}\frac{1}{2} \times \arg\left(1 - S^2\right) [\text{rad}] \\ &= -\frac{1}{2} \times \ln\left|1 - S^2\right| [\text{Np}] - \text{j}\frac{1}{2} \times \arg\left(1 - S^2\right) [\text{rad}] \end{aligned} \tag{C.95}$$

C.10.2 Scattering parameters and scattering matrix of a two-port

A two-port, shown in Figure C.24, can be treated as two individual one-ports, face-to-face.

For both one-ports, the incident and reflected waves are characterized by

$$\begin{aligned} V_{i1} &= \frac{1}{2} \times \left(\frac{U_1}{\sqrt{R_{01}}} + \sqrt{R_{01}}\, I_1\right) \\ V_{r1} &= \frac{1}{2} \times \left(\frac{U_1}{\sqrt{R_{01}}} - \sqrt{R_{01}}\, I_1\right) \\ V_{i2} &= \frac{1}{2} \times \left(\frac{U_2}{\sqrt{R_{02}}} + \sqrt{R_{02}}\, I_2\right) \\ V_{r2} &= \frac{1}{2} \times \left(\frac{U_2}{\sqrt{R_{02}}} - \sqrt{R_{02}}\, I_2\right) \end{aligned} \tag{C.96}$$

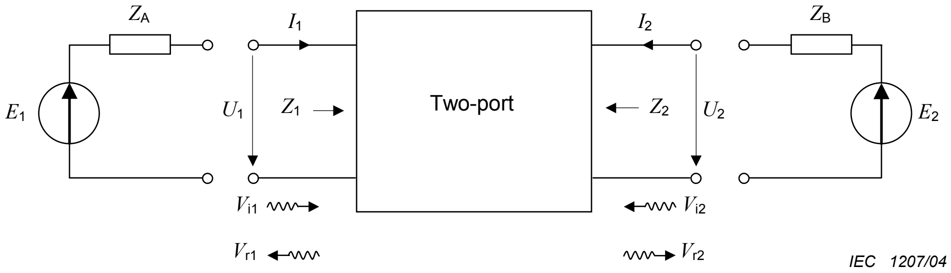

Figure C.24 – A two-port

Where R_{01} and R_{02} are the reference impedance at input and output, respectively, V_{i1} and V_{r1} are represented by the square roots of incident (unreflected) and reflected complex powers (see note) at port 1, and V_{i2} and V_{r2} are those at port 2.

NOTE Complex power is the product = UI. Apparent power is the product UI^*, which is used in electrical power technique, where the angle between the voltage and current is of interest. I^* is the complex conjugate of the current I.

The scattering parameters S_{mn} of a two-port are defined as follows:

$$\begin{aligned} V_{r1} &= S_{11}V_{i1} + S_{12}V_{i2} \\ V_{r2} &= S_{21}V_{i1} + S_{22}V_{i2} \end{aligned} \tag{C.97}$$

or in a matrix form

$$\begin{bmatrix} V_{r1} \\ V_{r2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix} \tag{C.98}$$

where the matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{C.99}$$

is called the scattering matrix [4]. It has the elements:

$$\begin{aligned} S_{11} &= \left.\frac{V_{r1}}{V_{i1}}\right|_{V_{i2}=0} = \frac{Z_1 - Z_A}{Z_1 + Z_A} = \rho_{B11} \\ S_{12} &= \left.\frac{V_{r1}}{V_{i2}}\right|_{V_{i1}=0} = \frac{2 \times U_1}{E_2}\sqrt{\frac{Z_B}{Z_A}} = T_{B12} \\ S_{21} &= \left.\frac{V_{r2}}{V_{i1}}\right|_{V_{i2}=0} = \frac{2 \times U_2}{E_1}\sqrt{\frac{Z_A}{Z_B}} = T_{B21} \\ S_{22} &= \left.\frac{V_{r2}}{V_{i2}}\right|_{V_{i1}=0} = \frac{Z_2 - Z_B}{Z_2 + Z_B} = \rho_{B22} \end{aligned} \tag{C.100}$$

with

$$\Gamma_B = 20 \times \log_{10} \frac{1}{T_B} \text{ [dB]} \tag{C.101}$$

The above refers to the quantities under operational conditions, i.e. to the complex operational reflection coefficient ρ_B and to the operational transfer function T_B (see Equation (C.56). They are directly derived from the condition that V_{i2} and V_{i1} are zero, a condition which is satisfied, in accordance with Equation (C.96) as soon as the terminal impedances Z_A and Z_B are equal to the reference impedance R_{01} and R_{02}, respectively. Hence,

S_{11} or ρ_{B11} is the complex operational reflection coefficient at the input;

S_{22} or ρ_{B22} is the complex operational reflection coefficient at the output;

S_{21} or T_{B21} is the operational transfer function in the forward direction;

S_{12} or T_{B12} is the operational transfer function in the backward direction.

The scattering matrix can thus be written in the form:

$$[S] = \begin{bmatrix} \rho_{B11} & T_{B12} \\ T_{B21} & \rho_{B22} \end{bmatrix} \tag{C.102}$$

$$S_{11} = \left. \frac{V_{r1}}{V_{i1}} \right|_{V_{i2}=0} \tag{C.103}$$

The connection between the scattering parameters and the above-mentioned working quantities can be derived as follows: the condition $V_{i2} = 0$ and its impact on the reflection factor at the input of the two-port is considered first.

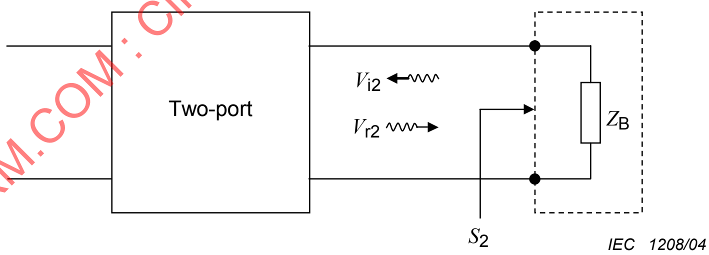

Figure C.25 – Termination Z_B by virtue of the scattering parameters of the two-port

Let us consider (see Figure C.25) the influence of the termination Z_B on the parameters V_{i2} and V_{r2} of the two-port. The scattering parameter of the termination is:

$$S_2 = \frac{Z_B - R_{02}}{Z_B + R_{02}} \tag{C.104}$$

where R_{02} is the reference impedance at the output of the two-port. The connection between V_{i2} and V_{r2} is obtained by the relation:

$$V_{i2} = S_2 V_{r2} \tag{C.105}$$